Armenian Journal of Physics, 2018, vol. 11, issue 3, pp. 125-129

Generation of Optical Radiation in Nanotube Undulators

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Received 01 June 2018

Abstract. The possibility to generate an sufficiently intense, monochromatic and directed radiation in the near ultraviolet range was investigated. Such radiation has important practical application n biology and medicine. It is shown that for a certain choice nanotube parameters channeled relativistic positrons in it the bunch will radiate as gamma photons and the "water window" photons also.

Keywords: tunable radiation, channeling, nanotube undulator, positron bunch, medium polarization, water window.

1. Introduction

We suppose that nanotube undulator has a spiral shape. Then channeled positrons, except oscillations in nanotube, on average, also moves along a spiral path and generates undulator radiation. The generated radiation by a relativistic bunch of oscillating charged particles has been predicted by Ginzburg [1]. In [2] Korhmazyan was considered this radiation, which called the undulator radiation by Motz [3], in the field of X-ray wavelengths. In [4] the characteristics of the undulator radiation in a dispersive medium were obtained. In the Lindhardt work [5] it explained the phenomenon of channeling of charged particles in crystals. The channeling radiation has been investigated by Kumakhov [6]. Additional radiation of the plane-channeled positrons due to oscillations with a period of sinusoidal curved crystal (crystal undulator (CO)) was considered in [7]. Study of the spectral characteristics of the radiation generated in the CO performed in [8] without taking into account polarization of the medium. In [9] these characteristics considering the polarization of the medium were obtained in the crystalline undulators (CO) and in the nanotube undulators (NO). The trajectories of channeled positrons in NO were obtained in [10]. In [11] it investigates the characteristics of the radiation generated in the dispersive medium of NU.

In this paper, the possibility of obtaining of a sufficiently intense directional quasi-monochromatic radiation is investigated in the ultraviolet wavelength region, namely in the "water window" region [12].

In Section 2, a brief description of the derivation of formula for the spectral distribution of the number of photons which are formed due to the relativistic positrons in the presence of the dispersive medium of the spiral nanoundulator. The frequency distribution of the radiation is given for the first harmonic of the radiation at zero angles.

In Section 3 for saving of the phenomenon of channeling positrons in NU is given the limitation of nanoundulator parameters.

The choice of parameters to form the directional beam of the optical photons, especially photons from the area "water window" performed in Section 4. Further it follows the conclusion (Section 5).

2. Obtaining a Formula for the Spectral Distribution of the Number of Photons

It is known that if the energy of emitted photons is much smaller than the energy of a relativistic bunch of charged particles, the radiation field can be found by methods of classical electrodynamics. This limitation on the wavelength of the radiation is $\lambda \gg \lambda_c / \gamma$, where λ_c is a Compton wavelength, γ is the energy of the positron in units of the rest energy of mc^2. The radiation field can be set by the integral along the trajectory of the particle by the interaction length L. The integral is taken from the two multipliers. The first multiplier is the cross product of the unit vector in the direction of the radiation and the particle velocity. The second multiplier is the phase function that contains the phase of the radiation field in the exponent. Due to the average motion of the channeling positron in the spiral nanoundulator, in the phase, contained in the scalar product of wave vector and the radius vector of the particle, appears an additional member, which varies according to a harmonic law. The exponential function of this multiplier can be decomposed in a series of the Bessel functions. The result is an additional phase, which ensures the law of conservation of energy-momentum in the radiation.

The spectral distribution of the number of photons produced by positron oscillating in NU with a spatial period l and radius R is the following:

$$\frac{d^2 N_{ph}}{d\lambda d(\cos\vartheta)} = \frac{2\pi\lambda L^2}{\lambda^3} \sum_{k=1}^{\infty} \left[\left(\sin\vartheta - k\frac{\beta_\perp}{\beta_\parallel a} \right)^2 J_k^2(a) + \beta_\perp^2 \dot{J}_k^2(a) \right] F(z_k), \tag{1}$$

$$z_k = \frac{\pi L}{\lambda}\left(1 - \beta_\parallel \sqrt{\varepsilon}\cos\vartheta - k\frac{\lambda}{l}\right), F(z_k) = \frac{\sin^2 z_k}{z_k{}^2}, a = \beta_\perp \frac{l}{\lambda}\sin\vartheta.$$

Here $\alpha = 1/137$ is the fine structure constant, λ and ϑ are the wavelength and the angle of radiation, k is the harmonic number, $\beta_\parallel$ and $\beta_\perp$ are the longitudinal and transverse particle velocities in units of speed of light c in vacuum. $J_k, \dot{J}_k$ are the Bessel function of k-th order and its derivative with respect to the argument, $F(z_k)$ is the diffraction sine, ε is a dielectric permittivity of NU. Ignoring the small speed change of particles β by radiation we have the following expression $\beta^2 = \beta_\parallel^2 + \beta_\perp^2$. NU parameter is defined with the following way $= \beta_\perp \gamma$. Taking into account these expressions $\gamma^{-1} = 1 - \beta^2$ and $\gamma_\parallel^{-1} = 1 - \beta_\parallel^2$ we have the following expression $\gamma^2 = \left(1 + q^2\right)\gamma_\parallel^2$. An interest is the generation of the optical photons with wavelengths that satisfy the condition $\lambda \ll \lambda_p$, where λ_p is the plasma wavelength of the medium. For such waves the dielectric permittivity is represented as

$$\varepsilon(\lambda) = 1 - \frac{\lambda^2}{\lambda_p^2}, \tag{2}$$

Moreover, note that the expression $F(0)=1$ is a consequence of the law of conservation of energy and momentum for the formation of radiation for the $k-$th harmonic. Since $\beta_{||}\leq 1,\quad \varepsilon\leq 1$ and $\lambda \ll l,$ then z_k becomes zero when $\vartheta \ll 1$. On the other hand, to get a bunch of optical photons emitted at an angle $\vartheta=0,$ we should to consider the radiation formed by the first harmonic $(k=1),$ because at zero angle the first harmonic is emitted by only. The spectral distribution of radiation of the first harmonic at very small angles $(a\ll 1)$ has the following form

$$\frac{d^2N_{ph}}{d\lambda d(\cos\vartheta)}=\frac{\pi\alpha q^2L^2}{4\gamma^2\lambda^3}\left[1+\left(1-\frac{l}{\lambda}\vartheta^2\right)^2\right]F(z), \tag{3}$$

$$z=\frac{\pi L}{2\lambda}\left[\vartheta^2-\frac{1}{\lambda_p^2}(\lambda_{\max}-\lambda)(\lambda-\lambda_{\min})\right].$$

where the first term of the expansion is leaving in the expansion of the Bessel function.

$$\lambda_{\substack{\min\\ \max}}=\frac{\lambda_p^2}{l}\left[1\mp\sqrt{1-\left(\frac{\gamma_{th}}{\gamma}\right)^2}\right],\gamma_{th}=\frac{l\sqrt{Q}}{\lambda_p},Q=1+q^2. \tag{4}$$

The radiation has an energy threshold, since, when $\gamma<\gamma_{th}$ the radiation in the medium is not formed due to of its polarization. When the condition $\gamma=\gamma_{th}$ is satisfied, only one wavelength $\lambda=\lambda_p^2/l$ is radiated [4] if the interaction length is an infinite value.The radiation wavelength has a maximum value $2\lambda_p^2/l$ if $\gg\gamma_{th}$ .Then, with accuracy up to small order $((\gamma_{th}/\gamma)^2),$ we have the following expressions for the critical wavelength values

$$\lambda_{\min}=\frac{lQ}{2\gamma^2},\lambda_{\max}=\frac{2\lambda_p^2}{l}. \tag{5}$$

The spectral distribution of a zero angle for the wavelengths $\lambda_{\substack{\min\\ \max}}\,(F=1)$ has the following form

$$\left.\frac{d^2N_{ph}}{d\lambda d\vartheta^2}\right|_{\vartheta=0}=\frac{\pi\alpha q^2L^2}{2\gamma^2\lambda^3}. \tag{6}$$

The relative widths of the lines are determined from the condition

$$\left|\Delta z_0\right|=\pi, z_0=\pi n\left(1-\frac{\lambda}{\lambda_{\max}}\right)\left(1-\frac{\lambda_{\min}}{\lambda}\right), \tag{7}$$

and are equal to the following expressions

$$\frac{\left|\lambda_{\max}-\lambda\right|}{\lambda_{\max}}=\frac{\left|\lambda-\lambda_{\min}\right|}{\lambda_{\min}}=n. \tag{8}$$

where $n = L/l$ is the period number of NU. Then the total number radiated by channeled positrons into NU at a zero angle for extremely hard and soft photons are respectively equal to

$$\begin{aligned} N_{ph}(\lambda_{\max}) &= \frac{\pi\alpha n q^2}{Q}\frac{l\lambda_{\min}}{\lambda_{\max}^2}, \\ N_{ph}(\lambda_{\min}) &= \frac{\pi\alpha n q^2}{Q}\frac{l}{\lambda_{\min}}. \end{aligned} \tag{9}$$

3. Restrictions on the Parameters of the Carbon NU

The depth of the potential well of the carbon nanotube is $U_0 = 32eV$ and the plasmon energy of medium is $\hbar\omega_p = 31eV$ [10]. If the angle between the direction fall of the positron in nanotube and its axis is smaller than the angle Lindhardt $\theta_L = \sqrt{2\nu/\gamma}$, that the channeling phenomena is takes place (ν is the potential well depth).

Let the positron bunch falls under the zero angle in a spiral undulator consisting of closely packed carbon nanotubes. If the angle of the helix deviation from its axis $2\pi R/l$ is less Lindhardt angle, the channeling condition will conserved. Thus, the parameters of NU is limited by the condition

$$\frac{R}{l} < \frac{1}{2\pi}\sqrt{\frac{2\nu}{\gamma}}$$

or

$$q < q_{ch}, q = \frac{2\pi R\gamma}{l}, q_{ch} = \gamma\theta_L = \sqrt{2\nu\gamma}. \tag{10}$$

4. The Choice of Parameters of NU

The spatial period of NU is chosen according to (5), and the radius of the helix is determined by the condition (10):

$$l = \frac{2\lambda_p^2}{\lambda_{\max}}, R < \frac{l}{2\pi}\sqrt{\frac{2\nu}{\gamma}}. \tag{11}$$

The positron bunch chosen with such energy that the parameter q was greater or the order of unity, as the number of radiated photons is proportional to q^2 .

Using the positron bunch with the LCLS parameters: the positron number in bunch is $E = 13.6GeV(\gamma = 2.66 \cdot 10^4)$, $N_b = 1.56 \cdot 10^9$, the positron energy (bunch energy) is, the transverse section is $S = 1.18 \cdot 10^6 cm^2 (\sigma_r = 6.12 \cdot 10^{-4} cm)$, that the channeling parameter is equal $q_{ch} = 1.82$.

To generate photons with wavelength $\lambda_{\max} = 2.4nm$ ("water window" region: $2.3 \div 4.4nm$ or $0.54KeV \div 0.28KeV$), it is necessary to create NU with $l = 1.3\mu m$ and $R = 0.8\overset{0}{A}\left(q = 1.8\right)$. Then, using formulas (5) and (9) for the total number of photons emitted by positron bunch in NU we obtain

$$N_{tot} = 1.2 \cdot 10^{-5} n. \tag{12}$$

Then, on the interact length $L = 0.75cm(n = 5600)$ will be generated the beam consisting of $N_{tot} = N_b N_{ph} = 10^8$ photons with the energy $\hbar\omega_{\min} = 0.52KeV(\lambda_{\max} = 2.4nm)$ and with the relative energy spread $1.8 \cdot 10^{-4}$. In the result we will obtain the directional beam at the zero angle with the surface density $N_{tot}/S = 8.5 \cdot 10^{13} cm^{-2}$.

5. Conclusion

The carbon nanotube undulator with the certain parameters can serve as the compact radiator for the generation of optical photons, using the relativistic positron bunch. It was shown the possibility of generation of the monochromatic, intense and directed photon beam with energy from the “water window” region. Such beams of photons are necessary for the biological research and for the practical application in medicine.

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