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The Dynamics of Instability Development in Spatially Separated Beam-Plasma System

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Abstract: The instabilities in system consisting of electron beam and spatially separated dissipative plasma is investigated in details. A study is developed that allows carrying out the investigation in general form, independently on system geometry, specific parameters, external fields etc. It only is assumed slight overlapped of the beam and the plasma fields (so-called weak beam-plasma coupling). In this case the beam-plasma interaction significantly differs from conventional case of full overlap (strong coupling). Under weak coupling the role of the beam proper oscillations becomes decisive. The instability is due to growth of the beam negative energy wave. With increase in level of dissipation the instability turns to a new, up to recently unknown type of dissipative instability.

Keywords: Spatially separated beam-plasma system, electron beam, dissipative instability.

1. Introduction

Very vast literature has grown on beam-plasma interaction since discovering of the instability of low density electron beam passing through plasma. This is largely related to the idea of design of high power sources of electromagnetic radiation based on this phenomenon. It was assumed that the sources would have many advantages as compared to well-known vacuum devices [1-2]. Further it becomes clear that spatial separation of the beam and the plasma in waveguide provides better conditions for the sources. Modern sources correspond to cylindrical waveguide with thin annular plasma and spatially separated thin annular electron beam [1-2]. In this case the operation of the sources is based on the beam interaction with the surface wave of hollow plasma cylinder. In contrary to case of strong beam-plasma coupling when the beam and the plasma fields are fully overlapped in opposite case of weak coupling the role of the beam oscillations are crucial. The beam-plasma instability becomes due to growing of the negative energy wave (NEW) of the beam. In this connection the role of all factors those lead to excitation of the NEW increases also and they should be taken into account. Among them dissipation plays most important role [3].

Present investigation considers the problem of time evolution of initial perturbation under weak beam-plasma coupling i.e. when the beam and the plasma fields are overlapped slightly (weak coupling). Developing fields are presented in form of wave train with slowly varying amplitude (SVA). It is shown that the evolution of SVA is described by partial differential equation of second order. The equation is solved and the space time structure of the field is analyzed. The solution of the equation gives very much and complete information on the instability, most of which is unavailable by other methods. The expression for the fields' evolution explicitly shows that with increase in level of dissipation the instability transforms to the new type of dissipatice instability [4]

2. Statement of the problem. Dispersion relation.

From elcctrodynamical point of view spatially separated beam-plasma system is nothing else as multilayer structure. Traditional analytical consideration of such systems encounters certain difficulties. It leads to a very cumbersome dispersion relation (DR), which, (i) complexly depends on geometry and (ii) sharply complicates with increase in number of layers. However, in the case of slight overlap of the beam and the plasma fields the problem may be considered by perturbation theory, based on smallness of coupling parameter. It leads to the dispersion relation (DR) [4,5]

$$D_p(\omega,k)D_b(\omega,k)=G\left(\kappa^4\delta\varepsilon_p\delta\varepsilon_b\right)_{\omega=\omega_0,k=k_0} \tag{1}$$

where $D_{p,b}(\omega,k)=k_{\perp p,b}^2-\kappa^2\delta\varepsilon_{p,b}$, $\kappa^2=k^2-\omega^2/c^2$, $\delta\varepsilon_p=\omega_p^2/\omega(\omega+i\nu)$, $\delta\varepsilon_b=\omega_b^2/\gamma^3(\omega-ku)^2$, $\omega_{p,b}$ are the Lengmuir frequencies of the beam and the plas and G is the small coupling coefficient [2,3]. The DR obviously shows interaction of the beam and the plasma waves. The interaction is weak. The DR (1) is carried out based actually on single assumption: weak beam-plasma coupling. It leads to following equation for determination of the growth rates (GR) of developing instabilities. The instability is due to interaction of NEW with plasma wave. The GR attains its maximum in resonance of NEW with plasma wave Resonance of this type (wave-wave) named Collective Cherenkov resonance [6]. The equation for GR is

$$\left(x'+i\nu/(2\gamma^2ku)\right)x'=-G\sqrt{\alpha}/4\gamma^3 \tag{2}$$

where x' is the deviation from the exact expression of NEW. In absence of dissipation the growth rate of instability caused by NEW growth is

$$\delta_{\text{NEW}}^{(\nu=0)}=(ku/2\gamma)\sqrt{(G/\gamma)\sqrt{\alpha}} \tag{3}$$

Dissipation exhibits itself as additional factor that intensifies growth of the NEBW. Equation (2) gives following expression for the growth rate upon arbitrary level of the dissipation [6].

$$\delta(\lambda)=\delta_{\text{NEW}}^{(\nu=0)}f(\lambda) \qquad f(\lambda)=\sqrt{1+\lambda^2/4}-\lambda/2 \qquad \lambda=\left(1/2\gamma^2\right)\left(\nu/\delta_{NEW}^{(\nu=0)}\right) \tag{4}$$

The expression (4) obviously shows the growth rate upon increasing in level of dissipation. In limit of strong dissipation $\lambda\to\infty$, it presents maximal growth rate of the new type of dissipative instability, shown up in [6] (It also follows from (2) by neglecting first term in parentheses)

$$\delta_{\text{NEW}}^{(\nu\to\infty)}=\left(G\sqrt{\alpha}/2\gamma\right)(ku)^2/\nu\equiv2\gamma^2\left(\delta_{\text{NEW}}^{(\nu=0)}\right)^2/\nu \tag{5}$$

3. The space-time evolution of the instability and its transition to dissipative type.

Now our aim is based on the DR (1) to solve the problem of initial pulse time evolution [7]. Let an instant $t=0$ in point $z=0$ a perturbation arises and the instability begins developing. We present developing fields form wave train of following type

$$E(z,t)=E_0(z,t)\exp\{-i\omega_0t+ik_0z\} \tag{6}$$

where ω_0 and k_0 satisfy the conditions $D_b(\omega,k)=0$ and $D_p(\omega,k)=0$. We also assume that the amplitude $E_0(z,t)$ is slowly varying as compared to ω_0 and k_0 i.e.

$$|\partial E_0/\partial t|<<\omega_0E_0; \qquad |\partial E_0\partial z|<<k_0E_0 \tag{7}$$

The behavior e slowly varying amplitude (SVA) $E_0(z,t)$ is actually governed by the DR. To derive the equation for SVA we recall that the fields vary near ω_0 and k_0 and use formal substitutions

$$\omega \to \omega_0 + i\frac{\partial}{\partial t}; \qquad k \to k_0 - i\frac{\partial}{\partial z} \tag{8}$$

Expanding the DR (1) in power series near ω_0 and k_0 one can obtain following equation for SVA

$$\left(\frac{\partial}{\partial t} + \mathrm{v}_b\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial t} + \mathrm{v}_\mathrm{p}\frac{\partial}{\partial z} + \nu^*\right)E_0(z,t) = \delta_0^2 E_0(z,t) \tag{9}$$

where $\mathrm{v}_{p,b}$ are group velocities of the plasma wave and the NEW of the beam respectively, $\mathrm{v}_b > \mathrm{v}_p$, $\delta_0 \equiv \delta_{\mathrm{NEW}}^{(\nu=0)}$ and $\nu^* = \operatorname{Im} D_p\left(\partial D_p/\partial\omega\right)^{-1}$ is proportional to collision frequency $\nu^* = const\cdot\nu$.

The equation may be solved by using Fourier transformation with respect to coordinate z and Laplace transformation with respect to time t. Corresponding inverse transformation

$$E_0(z,t) = \frac{1}{(2\pi)^2}\int\limits_{C(\omega)} d\omega \int\limits_{-\infty}^{\infty} \frac{dk \exp(-i\omega t + ikz)}{(\omega - ku)(\omega - k\mathrm{v}_0 + i\nu) + \delta_0^2} J(\omega,k) \tag{10}$$

gives needed result. Here the function $J(\omega,k)$ is determined by initial conditions. Its specific form of this function is not essential for following. In (10) $C(\omega)$ is the contour of integration over ω. It is a straight line that lies in the upper half plane of the complex plane $\omega = \operatorname{Re}\omega + i\operatorname{Im}\omega$ and passes above all singularities of the integrand. Thus, the problem has been reduced to the integration in (10). For convenience we transform the integration variables to another pair $\omega, k \to \omega, \omega' = \omega - ku$. We carry out the first integration (over ω) by residue method, but the second – by steepest descend method. As a result we have the expression for SVA

$$E_0(z,t) = -\frac{J_0}{2\sqrt{\pi}}\frac{\exp \chi_\nu^{(ss)}(z,t)}{\left(\mathrm{v}_\mathrm{b} - \mathrm{v}_p\right)^{1/2}\delta_0^{1/2}\left(\mathrm{v}_\mathrm{b} t - z\right)^{1/2}} \tag{11}$$

$$\chi_\nu^{(\mathrm{ss})} = \frac{2\delta_0}{\mathrm{v}_\mathrm{b} - \mathrm{v}_p}\sqrt{(z - \mathrm{v}_p t)(\mathrm{v}_b t - z)} - \nu\frac{\mathrm{v}_b t - z}{\mathrm{v}_\mathrm{b} - \mathrm{v}_p}:$$

$$J_0 = J\left(\omega = \omega(\omega_s'), \omega' = \omega_s'\right).$$

4. Analyses of the dynamics

At first glance we have arrived to complicate expression. However, the expression (11) lends itself to simple analysis. The analysis gives many information on the instability and its dynamics: its convective character, growth rates (spatial and temporal), velocities of unstable perturbations, influence dissipation on it. This information composes overall picture of the instability and dynamics its development. The structure of (11) explicitly shows that the space distribution and development dynamics of the fields is given mainly by the factor

$$\sim \exp \chi_\nu^{(ss)} \tag{12}$$

The analysis of the expression for $\chi_\nu^{(ss)}$ shows that the front of induced waveform moves at beam velocity v_b, but the back edge – at velocity v_p , (see curve 1 in Fig 2) i.e. velocities υ of unstable perturbation vary through the range $\mathrm{v}_p \le \upsilon \le \mathrm{v}_b$. This explicitly shows the convective character of the instability only by determination and without reference on complicate rules. The expression $\frac{\partial}{\partial z}\chi_\nu^{(ss)} = 0$ gives the place and velocity of the peak. In absence of dissipation it moves at average velocity $z_{pk} = \mathrm{v}_{pk} t$ and places at the middle of induced waveform at all instants $\mathrm{v}_{pk} = (\mathrm{v}_b + \mathrm{v}_p)/2$. Developing waveform is fully symmetric with respect to its peak at all instants. Substitution of the peak coordinate into $\chi_\nu^{(ss)}$ gives the field value in the peak. It exponentially increases and the growth rate is equal to $\delta_0 \equiv \delta_{\mathrm{NEW}}^{(\nu=0)}$ (3). The well-known initial problem gives the same maximal growth rate but it cannot specify place, where the most intensive growth occurs. The coincidence to the results of initial problem testifies presented approach: its initial assumptions, mathematics and results. It may appear that this way of analysis is a bit more complicate. However, it must be admitted that along with growth rates we have obtained much additional information. The information obviously clarifies the picture of the instability development and makes it realistic. The results of the boundary problem also follow from the expression for $\chi_\nu^{(ss)}$ (see below). One can easily see the merits of presented approach. In a fixed point z the field first increases increases up to value $E_0 \sim \exp \delta_0 z/(\mathrm{v_p v}_b)^{1/2}$.

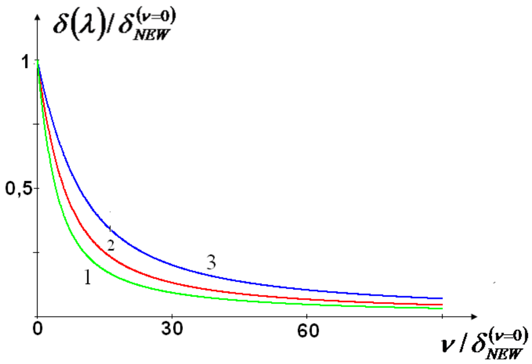

Fig 1. Dependence of the growth rate on dissipation (see (4))

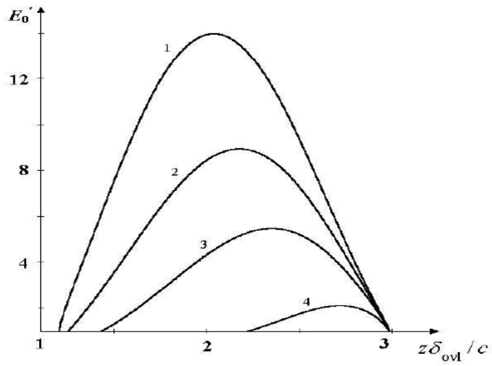

Fig 2. Shapes of the waveform vs longitudinal coordinate at fixed instant $t = 3/\delta_0$ for various values of parameter $k = \nu/\delta_0$, $k_1 = 0, k_2 = 1, k_3 = 2, k_4 = 4$

This maximum is attained at the instant $t = z/\mathrm{v}_a$ where $\mathrm{v}_a = 2\mathrm{v}_\mathrm{p}\mathrm{v}_b/(\mathrm{v}_\mathrm{p} + \mathrm{v}_b)$. Then field falls off and at the time $t \ge z/\mathrm{v}_p$ the train passes given point. The velocity v_a is the group velocity of the resonant wave upon amplification calculated by DR (8). The exponent $\delta_0/(\mathrm{v}_\mathrm{p}\mathrm{v}_\mathrm{b})^{1/2}$ in (28) coincides to solution of the boundary problem and is the maximal spatial growth rate. This coincidence once more actually justifies all steps of presented approach.

Dissipation changes described dynamics of induced waveform. Growth rate in the peak as well as growth rates of all perturbations falls down. Dissipation suppresses slow perturbations. The threshold velocity is

$$\mathrm{V}_{th}^{(\mathrm{ss})} = (\lambda \mathrm{v}_\mathrm{b} + \mathrm{v}_\mathrm{p})/(1+\lambda); \qquad \lambda = \nu^2/4\delta_0^2 \tag{13}$$

The wave train shortens. Only high velocity perturbations (at velocities in the range $V_{th} < \upsilon < \mathrm{v}_b$) develop.

The expression for $\chi_\nu^{(\mathrm{ss})}$ gives not only the maximal growth rates (spatial and temporal). It also gives growth rate of perturbations moving at given velocity υ. Substitution of $z = \upsilon t$ leads to $E_0 \sim \exp\Gamma_0(\upsilon)t$ where

$$\Gamma_0(\upsilon) = 2\delta_{\mathrm{ovl}} \frac{\sqrt{(u-\upsilon)(\upsilon - \mathrm{v}_0)}}{u - \mathrm{v}_0} \tag{14}$$

For given velocity the dependence of the growth rates on the dissipation level takes following form

$$\Gamma_0(\upsilon) \to \Gamma_\nu(\upsilon) = 2\delta_{\mathrm{ovl}} \frac{\sqrt{(u-\upsilon)(\upsilon - \mathrm{v}_0)}}{u - \mathrm{v}_0} - \nu \frac{\mathrm{v}_b t - z}{\mathrm{v}_b - \mathrm{v}_p} \tag{15}$$

The dynamics of the peak in presence of dissipation may be obtained by analyzing the equation for peak accounting for dissipation. It takes following form

$$(z - \mathrm{v}_{pk}t)^2 - \lambda(\mathrm{v}_\mathrm{b}t - z)(z - \mathrm{v}_\mathrm{p}t) = 0 \tag{16}$$

The solution of this equation actually presents the peak's movement depending on level of dissipation

$$z_{\mathrm{pk}} = \mathrm{v}_{go}t\left\{1 + \sqrt{\frac{\lambda}{1+\lambda}\left(1 - \frac{\mathrm{v}_\mathrm{b}\mathrm{v}_0}{w_{go}^2}\right)}\right\} \tag{17}$$

This expression shows that actually the peak shifts to the front of developing waveform. This is a result of suppression of slow perturbations. Substitution of z_{pk} into $\chi_\nu^{(ss)}$ gives the maximal growth rate under arbitrary level of dissipation

$$E_0(z = z_m, t) \sim \exp(\delta_0 t \cdot f(\lambda)), \tag{18}$$

where the function $f(x)$ coincides to that presented in (4). Accordingly, in limit of high level dissipation the new type of dissipative instability develops and we have $E \sim \exp \delta_{NEBW}^{\nu\to\infty} t$,

where $\delta_{NEBW}^{\nu\to\infty}$ is given by (5). This obviouly show gradual transition between two type instabilities those develop in spatially separated beam-plasma system.
The shapes of induced waveform for various level of dissipation are plotted in Fig.2, while the dependence of maximal growth rate on dissipation level is plotted on Fig 1.

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