Armenian Journal of Physics, 2019, vol. 12, issue 3, pp. 226-232

Modeling of dc SQUIDs With Topologically Nontrivial Barriers

(Proceedings of the Int. Conference on "The Problems of Modern Condensed Matter Physics")

E. M. Benecha[1], I. R. Rahmonov[2,3], Yu. M. Shukrinov[2], A. E. Botha[4], N. Ananikian[5]

[1]*College of Science, Engineering and Technology, University of South Africa, P.O. Box 392, UNISA 0003, Pretoria, South Africa*
[2]*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, 141980, Russia*
[3]*Umarov Physical Technical Institute, TAS, Dushanbe, 734063 Tajikistan*
[4]*Dubna University, Dubna, 141980, Russia*
[5]*Department of Physics, University of South Africa, Florida, 1710, South Africa*
[5]*Alikhanyan National Science Laboratory, Alikhanian Br. 2, 0036 Yerevan, Armenia*

E-mail: shukrinv@theor.jinr.ru

Received: 30 June 2019

Abstract: We study the behavior of DC-SQUIDs with topologically nontrivial barriers and compare the results with the trivial case. The current voltage characteristics and critical current as a function of the external magnetic flux for the dc SQUID with trivial and nontrivial barriers is calculated. We have shown the shift of the so-called beating solutions at the resonance frequency of DC-SQUID with nontrivial barriers. We find that increasing the number of Josephson junctions N reduces the size of the voltage steps, with the effect more pronounced in the case of SQUIDs containing nontrivial barriers. Therefore, tuning N constitutes a convenient way of changing the junction parameter β_c , which can in turn be used to optimize the Majorana character of SQUIDS. These results are important from the experimental perspective since they provide a way of detecting and manipulating the Majorana bound states.

Keywords: dc-SQUID, Josephson junctions, Majorana Fermion, trivial barriers, nontrivial barriers.

1. Introduction

The possibility of creating the Majorana quasiparticles [1] in condensed-matter systems has become a focus of intense attention in recent years in view of their possible application as qubits in quantum computers [2]. Superconducting Quantum Interference Devices (SQUIDs) with Josephson junctions containing topologically nontrivial barriers are among the leading candidates for the detection and manipulation of the Majorana fermion [3,4]. The appearance of Majorana bound states in the superconducting junctions enables tunneling of quasiparticles with charge e across the junction, instead of $2e$ for Cooper pair transport in conventional barriers, resulting in the doubling of the Josephson periodicity of the tunneling current from $I_s = I_c \sin\varphi$ to $I_s = I_c \sin(\varphi/2)$. The doubled periodicity is predicted to lead to a SQUID modulation period $2\Phi_0$ of $I_s = I_c \sin\varphi$ instead of the usual Φ_0 periodicity, where $\Phi_0 = \dfrac{h}{2e}$ is the magnetic flux quantum, which in turn doubles the period of the flux dependence of the supercurrent from 2π to 4π [3].

The experimental detection of Majorana fermions in superconducting junctions containing topologically nontrivial barriers has mainly relied on measurement of zero-bias conductance peak [5,6,7] or observation of the 4π periodic dependency of the Josephson supercurrent [8], among other methods [9]. However, the effectiveness of the former set of experiments has been questioned since the expected $\frac{2e^2}{h}$ value of tunneling conductance zero-bias peak may be obscured by resonances from subgap states at nonzero energy as well as the presence of other effects such as magnetic impurities [10]. On the other hand, relaxation to equilibrium states, quantum phase slips, and the large bulk shunt present in contacts with topological insulators may reduce the 4π-periodic dependence. Therefore, there is need to find more definite signatures of the Majorana fermion in condensed matter systems.

Previously [11], it has been shown that dc-SQUIDs containing trivial barriers exhibit resonance behavior on their IV-characteristics, while a recent study [8] carried out on SQUIDs with topologically nontrivial barriers has demonstrated that the SQUID parameters β_L and β_c can be used to increase the ratio of Majorana tunneling to standard Cooper pair tunneling by more than two orders of magnitude. In Ref. [12] the phase dynamics of a SQUID consisting of two single Josephson junctions with topologically nontrivial barriers has been studied. It has been demonstrated that the branch of the current–voltage characteristic corresponding to the resonance frequency in the case of the SQUID with nontrivial barriers is displaced by $\sqrt{2}$ over voltage. The magnetic field dependence of return current of IV-curve for DC-SQUID with topologically nontrivial barriers taking into account the two component superconducting current (2π periodic Cooper pairs current and 4π periodic Majorana fermions current) has been studied in Ref.[13]. It has been shown that in case of two-component superconducting current the periodicity of magnetic field dependence of return current displaced by Cooper pairs and Majorana fermion ratio over the magnetic field. In Ref.[14] have been demonstrated the manifestation of parametric resonance and the excitation of a longitudinal plasma wave in a system of coupled Josephson junctions with topologically trivial and nontrivial barriers. The position of minimum of resonance voltage depending on the dissipation parameter for the system with nontrivial barriers is shifted from that for the system with trivial barriers. The Josephson junction (JJ) in the presence of the Majorana bound states exhibits $4\pi-$periodic oscillations of superconducting current [15], and the current - voltage (I-V) characteristic of such a JJ demonstrates only even Shapiro steps in contrast to its trivial analogs. This property was called the fractional Josephson effect and was actively investigated in recent years in various systems, because it may serve as an experimental evidence of the formation of such fermions. The properties of a Josephson junction with the $2\pi-$ and $4\pi-$periodic superconducting current component have been analyzed in Ref. [16]. In the range of low voltages, such a junction exhibits the 4π periodicity of the phase difference for the Majorana current amplitude much smaller than the Josephson current, which makes it possible to observe Josephson current oscillations with a fractional period for small dissipation $\beta < 1$ in the hysteresis region. The effect the $4\pi-$periodic Majorana current component is also manifested in a change in the sequence of steps in the ladder structure emerging on the current–voltage (I–V) characteristic of the junction.

In this work, we study in detail the behavior of dc-SQUID containing two stacks of JJ with topologically nontrivial barriers and carry out a comparative analysis with dc-SQUID with trivial barriers (where the Josephson junction is typically an ordinary metal or insulator) using the resistively and capacitively shunted Josephson junction (RCSJ) model. We find that the I-V characteristics between the two cases presents significant distinctions which might provide strong signatures in the experimental detection of the Majorana fermions.

2. Model and Methods

Charge transport through a Josephson junction containing a topologically nontrivial barrier is characterized by the standard Cooper pair tunneling $[\sin(\varphi)]$ and single electron tunneling by virtue of the Majorana fermion $[\sin(\frac{\varphi}{2})]$. The relative contribution of these two processes is determined by a factor γ which is connected to the charge carrier $q=\frac{2e}{\gamma}$. In this study, we consider two scenarios where the superconductor is either nontrivial $\gamma=2$ or topologically trivial $\gamma=1$

When a magnetic field threads a superconducting loop, the flux is quantized according to

$$\frac{\varphi_1-\varphi_2}{\gamma}+2\pi\frac{\Phi_t}{\Phi_0}=2\pi n$$

where $\varphi_1-\varphi_2$ is the phase drop over the junctions. The total flux Φ_t (normalized to the flux quanta $\Phi_0=h/2e$) constitutes the sum of the external flux Φ_e and the self-flux induced by the current flowing through the ring, and is given by

$$\frac{\Phi_t}{\Phi_0}=\frac{\Phi_e}{\Phi_0}+\beta_L(\chi_1-\chi_2)$$

where $\beta_L=\frac{2\pi}{\Phi_0}LI_c$ is a screening parameter which represents the ratio of the magnetic flux generated by the maximum possible circulating current I_c and $\frac{\Phi_0}{2\pi}$. The factor $\chi_{1,2}$ denotes the current dependence on the phase difference of the individual junctions and is given by $\chi_i=\alpha_i\sin(\varphi_i)+(1-\alpha_i)\sin(\varphi_i/2)$ with $\alpha=0$ for single electron tunneling due to the Majorana fermion (nontrivial case) or $\alpha=1$ due to Cooper pair tunneling (trivial case).

We have modelled the SQUID transport equations using the resistively and capacitively shunted junction (RCSJ) model as shown Figure 1.

Assuming an ideal Josephson junction shunted by a capacitor C and resistor R, the currents through the junctions can be written as $I=C\frac{dV}{dt}+\frac{V}{R}+I_c\chi_{1,2}$, where the voltage is related to the time derivative of the phase by the Josephson relation

$$V=\frac{\gamma\hbar}{2e}\frac{d(\varphi/\gamma)}{dt}=\frac{\hbar}{2e}\frac{d\varphi}{dt}$$

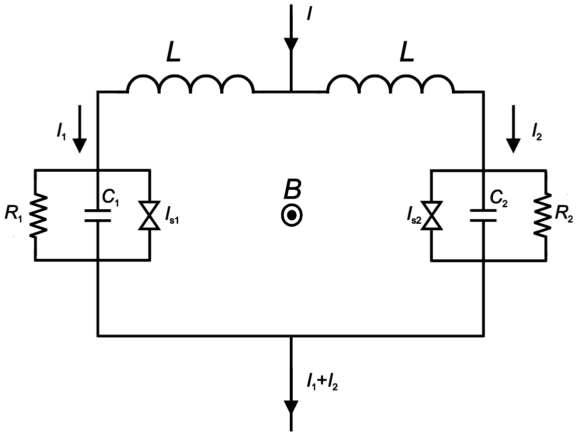

Figure 1. SQUID's equivalent circuit.

Therefore, the current through each junction can be written as

$$\begin{cases} I_1 = \beta_c \dfrac{d^2\varphi}{dt^2} + \dfrac{d\varphi}{dt} + \chi_1 \\ I_2 = \beta_c \dfrac{d^2\varphi}{dt^2} + \dfrac{d\varphi}{dt} + \chi_2 \end{cases}$$

where $\beta_c = 2\pi I_c R^2 C / \Phi_0$ is the Mc-Cumber parameter.

Applying the fluxoid quantization condition (equation 1), the voltage state can be described by the following differential equations

$$\begin{cases} \dfrac{\partial\varphi_1}{\partial t} = V_1 \\ \dot{V}_1 = \dfrac{1}{\beta_c}\left\{\dfrac{I}{2} - V_1 - \chi_1 + \dfrac{1}{2\beta_L}\left[2\pi(n\varphi_e) - \dfrac{1}{\gamma}\left(\sum_{i=1}^{N_1}\varphi_i \sum_{j=1}^{N_2}\varphi_j\right)\right]\right\} \\ \dfrac{\partial\varphi_2}{\partial t} = V_2 \\ \dot{V}_2 = \dfrac{1}{\beta_c}\left\{\dfrac{I}{2} - V_2 - \chi_2 \dfrac{1}{2\beta_L}\left[2\pi(n - \varphi_e)\dfrac{1}{\gamma}\left(\sum_{i=1}^{N_1}\varphi_i - \sum_{j=1}^{N_2}\varphi_j\right)\right]\right\} \end{cases}$$

which we have solved numerically using the forth order Runge-Kutta method.

3. Results and discussion

Figure 2 shows the I-V characteristics of a dc-SQUID with trivial and nontrivial Josephson barriers simulated for an external flux of $\Phi_e = 0$ and $\Phi_e = 1$. It can be seen that applying an external flux leads to the occurrence of branches in the I-V characteristics, so called beating solutions. The beating solutions occur when the Josephson frequency equals the frequency of the circuit formed by the loop inductance L and the junction capacitance C, in which case the resonance voltage becomes $V_{res} = n\sqrt{\frac{\gamma}{\beta_c \beta_L}}$, $(n = 1, 2, 3, ...)$, where $\gamma = 1$ and $\gamma = 2$ for trivial and nontrivial junctions, respectively. Importantly, we find that V_{res} for SQUIDs with nontrivial barriers is higher by a factor of $\sqrt{2}$ compared to SQUIDS with trivial barriers.

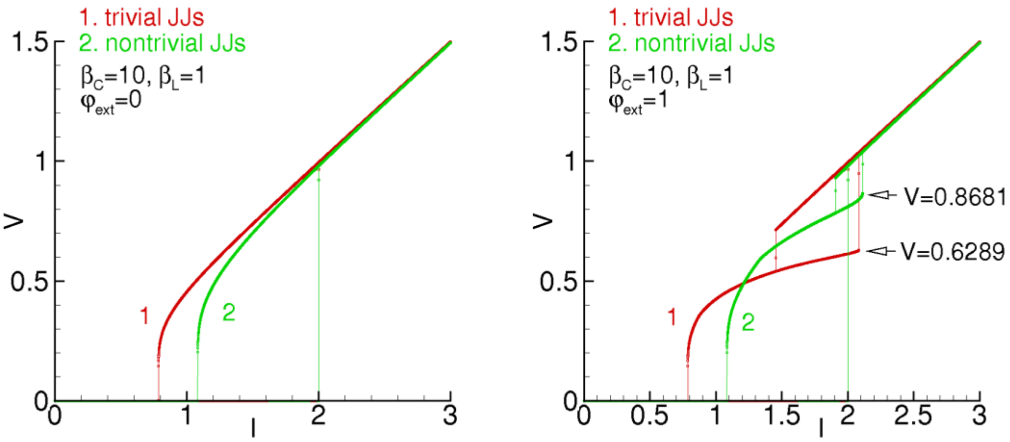

Figure 2. Current-voltage characteristics of a DC-SQUID with trivial and nontrivial Josephson barriers simulated for $\varphi_e = 1$; $\beta_c = 10$ and $\beta_L = 1$.

In order to understand the effect of increasing the number of junctions on the behavior of the SQUID, we have simulated the I-V characteristics for a stack of coupled $(\alpha = 0.1)$ Josephson junctions on each loop for both trivial and nontrivial cases, and the result is as shown in Figure 3.

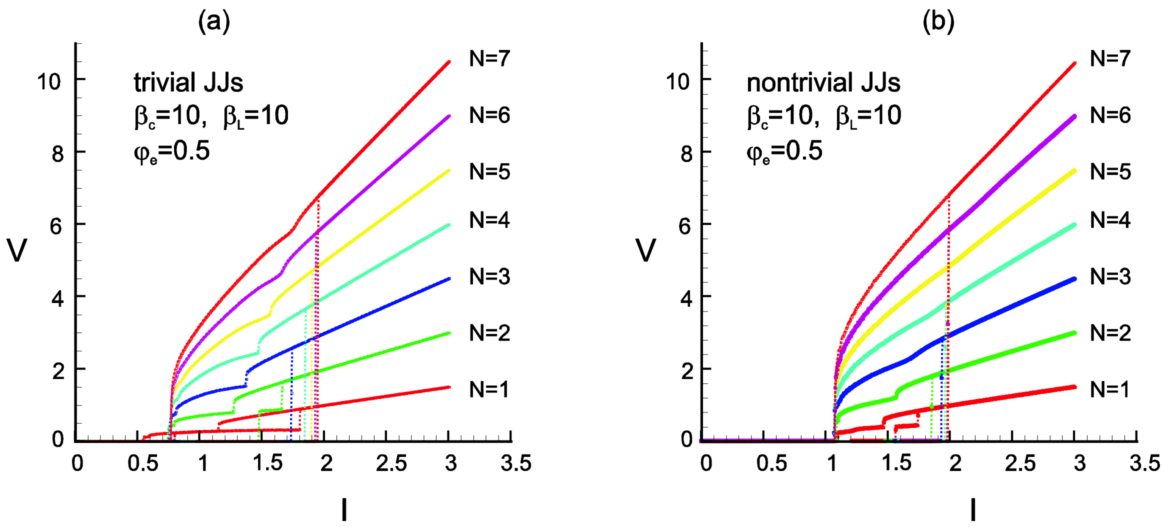

Figure 3. I-V characteristics of dc-SQUID with trivial and nontrivial Josephson junctions simulated for different number of coupled junctions, N=1,2,3,...,7. The interjunction coupling parameter is $\alpha = 0.1$

We find that increasing the number of Junctions reduces the size of the voltage steps, with the effect being more pronounced in the case of nontrivial barriers. This observation can be understood from the fact that total equivalent junction capacitance, given by $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_N}$ decreases with an increase in number of junctions, and is directly related to the SQUID parameter β_C. Therefore, for a SQUID of N junctions, the resonance voltage becomes $V_{res} = n\sqrt{\frac{2N}{\gamma_j \beta_c \beta_L}}$, which results in the resonance voltage step for nontrivial barriers ($\gamma_j = 1$), becoming smaller by factor of $\sqrt{2N}$ compared to trivial barriers ($\gamma_j = 2$). Since the junction capacitance is related to β_C, therefore, optimizing the number of junctions N may constitute a convenient way of changing the junction parameter β_C, which can in turn be used to optimize the Majorana character of SQUIDS [8].

4. Conclusions

We have carried out a comparative study of the I-V characteristics of dc SQUIDs containing either conventional (trivial) or topologically nontrivial Josephson junction barriers, and our results demonstrate that the I-V characteristics of the two are significantly different. Topological barriers support Majorana fermions, which are expected to be used for the realization of quantum gates that are topologically protected from local sources of decoherence. We find that the introduction of an external magnetic flux in the SQUID in both cases leads to the occurrence of additional branches in the I-V characteristics; however, with different resonance voltages. Importantly, we find that the resonance voltage in SQUIDs containing nontrivial Josephson junction barriers is higher by a factor of $\sqrt{2}$ compared to SQUIDS with trivial Josephson junctions. Further, we find that increasing the number of Josephson junctions N reduces the size of the voltage steps, with the effect being more pronounced in the case of SQUIDs containing nontrivial barriers. Therefore, tuning N constitutes a convenient way of changing the junction parameter β_C, which can in turn be used to optimize the Majorana character of SQUIDS [8]. These results are important from the experimental perspective since they provide a way of detecting and manipulating the Majorana bound states.

Acknowledgements

We thank Dr. D. V. Kamanin and South Africa – JINR collaboration for support this work.

Funding

Yu. M. S thank DAAD for support of his visit to DAAD School in Armenia, June 1-2, 2019.

Conflict of Interest

There is no conflict of interest.

Author Contributions

E. M. Benecha and I. R. Rahmonov conducted numerical simulations. Yu. M. Shukrinov and A. E. Botha participated in the writing of the text.

References

[1] E. Majorana, Nuovo Cimento **14** (1937) 171.
[2] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. **80** (2008) 1083.
[3] L. Fu and C. L. Kane, Phys. Rev. Lett., 100, 096407 (2008).
[4] Y. Tanaka, T. Yokoyama, and N. Nagaosa. Phys. Rev. Lett., 103, 107002 (2009).
[5] K. Sengupta, I. Zutic, H.J. Kwon , V.M. Yakovenko, and S. Das Sarma, Phys. Rev. B, 63 144531 (2001).
[6] C. J. Bolech and E. Demler, Phys. Rev. Lett., 98, 237002 (2007).
[7] K.T. Law , P.A. Lee, and T.K Ng, Phys. Rev. Lett., 103, 237001 (2009).
[8] M. Veldohorst, C. G. Molenaar, X. L. Wang, H. Hilgenkamp, and A. Brinkman, Appl. Phys. Lett., 100, 072602 (2012).
[9] C. W. J. Beenakker, Annu. Rev. Condens. Matter Phys., 4, 113 (2013).
[10] E. J. H. Lee, X. Jiang, R. Aguado, G. Katsaros, C. M. Lieber, and S. D. Franceschi, Phys. Rev. Lett., 109, 186802 (2012).
[11] W.-D. Schmidt, P. Seidel, and S. Heinemann, Phys. Stat. Sol., 91, K155 (1985).
[12] I. R. Rahmonov, Yu. M. Shukrinov and R. Dawood, JETP Letters, 103, 395 (2016).
[13] I. R. Rahmonov, Yu. M. Shukrinov, R. Dawood, H. El Samman, Low Temperature Physics 43, 824 (2017).
[14] I. R. Rahmonov, Yu. M. Shukrinov, K. V. Kulikov, T. Belgibaev, A. Plecenik, D. V. Anghel and W. Nawrocki, JETP Letters, 109, 33 (2019).
[15] M. Maiti, K. M. Kulikov, K. Sengupta, and Y. M. Shukrinov, Phys. Rev. B 92, 224501 (2015).
[16] K. V. Kulikov, R. Davud, E. P. Nakhmedov, and Yu. M. Shukrinov, Journal of Experimental and Theoretical Physics, 125, 333–339 (2017).