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Mathematics

ON SOLVABILITY OF A NONLINEAR DISCRETE SYSTEM IN THE SPREAD THEORY OF INFECTION

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In this paper a special class of infinite nonlinear system of algebraic equations with Teoplitz matrix is studied. The mentioned system arises in the mathematical theory of the spatial temporal spread of the epidemic. The existence and the uniqueness of the solution in the space of bounded sequences are proved. It is studied also the asymptotic behavior of the constructed solution at infinity. At the end of the work specific examples are given.

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Introduction and Statement of the Problem. In this paper we study the following infinite nonlinear algebraic system of equations

$$x_n = \sum_{j=-\infty}^{\infty} a_{n-j} g(x_j), \quad n \in \mathbb{Z}, \tag{1}$$

with respect to the unknown infinite vector $\mathrm{x} = (\dots, x_{-1}, x_0, x_1, \dots)^T$ (T denotes the transposition), where the conditions on the sequence $\{a_n\}_{n=0}^{\infty}$ and the function g are listed below.

We assume that infinite matrix $A = (a_{n-j})_{n,j=-\infty}^{\infty}$ satisfies the following conditions

$$a_n > 0, \quad \forall\, n \in \mathbb{Z}, \quad \sum_{n=-\infty}^{\infty} a_n = 1, \tag{2}$$

$$\sum_{n=-\infty}^{\infty} |n| a_n < +\infty, \quad \nu(A) = \sum_{n=-\infty}^{\infty} n a_n > 0, \tag{3}$$

and the function g satisfies:

$a)$ the function $g(u)$ is determined on the $\mathbb{R}^+$, is increasing on the interval $[0,\eta]$ and $g \in C[0,\eta]$;

$b)$ the function $g(u)$ is strictly-convex up on the interval $[0,\eta]$ and $g(0)=0$, $g(\eta)=\eta$;

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$c)$ there exist $\varepsilon > 0$ and $\tilde{c} > 0$ such that $g(u) \geq g'(0)u - \tilde{c}u^{1+\varepsilon}$, $u \in [0, \eta]$;

$d)$ $g'(0)$ is defined, $1 < g'(0) < +\infty$, and there is a number $\eta > 0$ such that $g(u) \leq g'(0)u$, $u \in [0, \eta]$.

The system (1) arises in the discrete diffusion model for the geographical spread of epidemics, as well as in the age-structured non-local delayed reaction-diffusion theory (see [1–5]).

In the case $a_{-n} = a_n$, this system (and its two-dimensional analogue) is well studied in the work [6], where alternating and bounded solutions are constructed under some restrictions on nonlinearity. In the present paper, under essentially different restrictions on the nonlinearity of g and on the matrix A, existence and uniqueness theorems of a positive and bounded solution are described. The asymptotic behaviour of the constructed solution at $\pm\infty$ is described. At the end of the paper specific examples of nonlinear g are given.

Auxiliary Facts. Consider the following discrete analogue of the Diekmann function (see [1])

$$L(\lambda) := g'(0) \sum_{j=-\infty}^{\infty} a_j q^{-\lambda j}, \quad q > 1, \quad \lambda \in [0, +\infty). \tag{4}$$

Hereinafter, we will assume the convergence of the series (4) and its termwise differentiability.

Observe that $L(0) = g'(0) \sum\limits_{j=-\infty}^{\infty} a_j = g'(0) > 1$. Under condition (3) we have

$$\frac{dL(\lambda)}{d\lambda}\Big|_{\lambda=0} = -g'(0) \sum_{j=-\infty}^{\infty} a_j j q^{-\lambda j} \cdot \ln q \Big|_{\lambda=0} < 0, \tag{5}$$

and due to the continuity of the function $\dfrac{dL(\lambda)}{d\lambda}$ there exists a number $\lambda_0 > 0$ such that

$$\frac{dL(\lambda)}{d\lambda} < 0 \quad \text{for any} \quad \lambda \in [0, \lambda_0].$$

From (5) it follows that

$$\frac{d^2L(\lambda)}{d\lambda^2} = g'(0) \sum_{j=-\infty}^{\infty} a_j j^2 q^{-\lambda j} \cdot (\ln q)^2 > 0 \quad \text{(it can also be } +\infty).$$

Therefore, the function $L(\lambda)$ is convex. We assume that

$$L(\lambda_0) < 1. \tag{6}$$

Hence, according to the Boltzano–Cauchy theorem, there exists a unique number $\sigma_0 \in (0, \lambda_0)$ such that

$$L(\sigma_0) = 1. \tag{7}$$

From the properties of the function $L(\lambda)$ it follows that

$$L(\sigma_0 + \delta) < 1 \tag{8}$$

for each $\delta \in (0, \lambda_0 - \sigma_0)$. Consider the following sequence

$$\mathcal{L}_n = \max\{\eta q^{\sigma_0 n} - M q^{(\delta+\sigma_0)n}, 0\}, \quad n \in \mathbb{Z}, \tag{9}$$

where $M > \eta$, $\delta \in (0, \lambda_0 - \sigma_0)$ are parameters. Notice that $\mathcal{L}_n = 0$, if $n \geq \dfrac{1}{\delta} \log_q \dfrac{\eta}{M}$, $n \in \mathbb{Z}$. It is easy to verify that for any $\delta \in (0, \min\{\lambda_0 - \sigma_0, \sigma_0 \varepsilon\})$ the inequality

$$\mathcal{L}_n^{1+\varepsilon} \leq \eta^{1+\varepsilon} q^{(\delta+\sigma_0)n}, \quad n \in \mathbb{Z}, \tag{10}$$

holds.

Solvability of System (1)**.** Consider the following iteration for system (1):

$$\begin{aligned} x_n^{(p+1)} &= \sum_{j=-\infty}^{\infty} a_{n-j} g(x_j^{(p)}), \quad n \in \mathbb{Z}, \quad p = 0,1,2,\dots, \\ x_n^{(0)} &= \begin{cases} \eta q^{\sigma_0 n}, & n \in \mathbb{Z} \setminus \mathbb{Z}^+, \\ \eta, & n \in \mathbb{Z}^+, \end{cases} \end{aligned} \tag{11}$$

where $\mathbb{Z}^+ := \{0, 1, 2, \dots\}$.

First of all observe that

$$x_n^{(p)} \downarrow \text{ with respect to } p, \quad n \in \mathbb{Z}, \tag{12}$$

$$x_n^{(p)} \geq 0, \quad n \in \mathbb{Z}, \quad p = 0,1,2,\dots \tag{13}$$

Let $n \in \mathbb{Z}^+$. Then, taking into account the monotonicity of the function g, as well as conditions $b)$, (2) and (3), from (11) we obtain

$$x_n^{(1)} \leq \sum_{j=-\infty}^{\infty} a_{n-j} g(\eta) = \eta = x_n^{(0)}, \quad n \in \mathbb{Z}^+.$$

Now let $n \in \mathbb{Z} \setminus \mathbb{Z}^+$. Then, taking into account conditions (2), (3), $a) - d)$, from (11) we get

$$\begin{aligned} x_n^{(1)} &= \sum_{j=-\infty}^{\infty} a_{n-j} g(x_j^{(0)}) = \sum_{j=-\infty}^{0} a_{n-j} g(x_j^{(0)}) + \sum_{j=1}^{\infty} a_{n-j} g(x_j^{(0)}) \\ &= \sum_{j=-\infty}^{0} a_{n-j} g(\eta q^{\sigma_0 j}) + \sum_{j=1}^{\infty} a_{n-j} g(\eta) \leq \eta g'(0) \sum_{j=-\infty}^{0} a_{n-j} q^{\sigma_0 j} + \eta \sum_{j=1}^{\infty} a_{n-j} \\ &= \eta g'(0) \sum_{i=n}^{\infty} a_i q^{\sigma_0 (n-i)} + \eta \sum_{i=-\infty}^{n-1} a_i \\ &= \eta g'(0) q^{\sigma_0 n} \left(\sum_{i=-\infty}^{\infty} a_i q^{-i\sigma_0} - \sum_{i=-\infty}^{n-1} a_i q^{-i\sigma_0} \right) + \eta \sum_{i=-\infty}^{n-1} a_i \\ &\leq \eta q^{\sigma_0 n} L(\sigma_0) - \eta q^{\sigma_0 n} \sum_{i=-\infty}^{n-1} a_i q^{-i\sigma_0} + \eta \sum_{i=-\infty}^{n-1} a_i \\ &\leq \eta q^{\sigma_0 n} - \eta \sum_{i=-\infty}^{n-1} a_i q^{\sigma_0} + \eta \sum_{i=-\infty}^{n-1} a_i \leq \eta q^{\sigma_0 n} = x_n^{(0)}. \end{aligned}$$

Hence, $x_n^{(1)} \leq x_n^{(0)}$ for $n \in \mathbb{Z} \setminus \mathbb{Z}^+$. It is obvious that $x_n^{(1)} \geq 0$, $n \in \mathbb{Z}$. Assuming that $x_n^{(p)} \leq x_n^{(p-1)}$, $n \in \mathbb{Z}$, and $x_n^{(p)} \geq 0$, $n \in \mathbb{Z}$, for some natural p, in light of (2) and the monotonicity of function g, from (11) we obtain

$$x_n^{(p+1)} \leq \sum_{j=-\infty}^{\infty} a_{n-j} g(x_j^{(p-1)}) = x_n^{(p)},$$

$$x_n^{(p+1)} \geq \sum_{j=-\infty}^{\infty} a_{n-j} g(0) = 0.$$

Thus statements (12) and (13) are proved.

Let us rewrite the successive approximation formula (11) in the form

$$\begin{aligned} x_n^{(p+1)} &= \sum_{i=-\infty}^{\infty} a_i g(x_{n-i}^{(p)}), \quad n \in \mathbb{Z}, \quad p = 0,1,2,\ldots, \\ x_n^{(0)} &= \begin{cases} \eta q^{\sigma_0 n}, & n \in \mathbb{Z} \setminus \mathbb{Z}^+, \\ \eta, & n \in \mathbb{Z}^+. \end{cases} \end{aligned} \tag{14}$$

Using the monotonicity of the zero approximation, on can easily check by induction that

$$x_n^{(p)} \uparrow \text{ with respect to } n, \quad n \in \mathbb{Z}, \quad p = 0,1,2,\ldots \tag{15}$$

We claim that for $M > \max\left\{\eta, \dfrac{\tilde{c}\eta^{1+\varepsilon} L(\delta+\sigma_0)}{g'(0)(1-L(\sigma_0+\delta))}\right\}$ and $\delta \in (0, \min\{\lambda_0-\sigma_0, \sigma_0\varepsilon\})$ it holds the inequality

$$x_n^{(p)} \geq \mathcal{L}_n \quad n \in \mathbb{Z}, \quad p = 0,1,2,\ldots \tag{16}$$

We will prove it by induction with respect to p.

The inequality (16) for $p = 0$ follows from the definition of the zero approximation. Assume that (16) is fulfilled for some natural p. Then, in view of conditions $a) - d)$, (2), (3) and (10), from (11) we obtain

$$\begin{aligned} x_n^{(p+1)} &\geq \sum_{j=-\infty}^{\infty} a_{n-j} g(\mathcal{L}_j) \geq g'(0) \sum_{j=-\infty}^{\infty} a_{n-j}\mathcal{L}_j - \tilde{c} \sum_{j=-\infty}^{\infty} a_{n-j}\mathcal{L}_j^{(1+\varepsilon)} \\ &\geq g'(0) \sum_{j=-\infty}^{\infty} a_{n-j}\left(\eta q^{\sigma_0 j} - M q^{(\sigma_0+\delta)j}\right) - \tilde{c} \sum_{j=-\infty}^{\infty} a_{n-j}\mathcal{L}_j^{(1+\varepsilon)} \\ &= \eta q^{\sigma_0 n} g'(0) \sum_{i=-\infty}^{\infty} a_i q^{-\sigma_0 i} - M q^{(\sigma_0+\delta)n} g'(0) \sum_{i=-\infty}^{\infty} a_i \eta q^{-(\sigma_0+\delta)i} - \tilde{c} \sum_{j=-\infty}^{\infty} a_{n-j}\mathcal{L}_j^{(1+\varepsilon)} \\ &\geq \eta q^{\sigma_0 n} L(\sigma_0) - M q^{(\sigma_0+\delta)n} L(\sigma_0+\delta) - \tilde{c} \sum_{j=-\infty}^{\infty} a_{n-j}\mathcal{L}_j^{(1+\varepsilon)} \\ &\geq \eta q^{\sigma_0 n} - M q^{(\sigma_0+\delta)n} L(\sigma_0+\delta) - \tilde{c}\eta^{(1+\varepsilon)} \sum_{j=-\infty}^{\infty} a_{n-j} q^{(\sigma_0+\delta)j} \\ &= \eta q^{\sigma_0 n} - M q^{(\sigma_0+\delta)n} L(\sigma_0+\delta) - \frac{\tilde{c}\eta^{(1+\varepsilon)}}{g'(0)} L(\sigma_0+\delta) q^{(\sigma_0+\delta)n} \\ &\geq \eta q^{\sigma_0 n} - M q^{(\sigma_0+\delta)n}. \end{aligned}$$

Thus it follows from (12), (13), and (16) that the sequence of vectors $\{\mathrm{x}^{(p)}\}_{p=0}^{\infty}$ (where $\mathrm{x}^{(p)} = (\ldots, x_{-1}^{(p)}, x_0^{(p)}, x_1^{(p)}, \ldots)^T$) has a limit as $p \to +\infty$. Namely,

$$\lim_{p\to\infty} \mathrm{x}^{(p)} = \mathrm{x}. \tag{17}$$

So the limit vector $\mathrm{x} = (\dots, x_{-1}, x_0, x_1, \dots)^T$ satisfies the initial system of (1), and the coordinates of the limit vectors satisfy

$$\mathcal{L}_n \le x_n \le x_n^{(0)}, \quad n \in \mathbb{Z}. \tag{18}$$

From (15) it follows that the coordinates x_n, $n \in \mathbb{Z}$, of the limit vectors x are increasing with respect to $n \in \mathbb{Z}$. Observe that

$$\lim_{n\to-\infty} x_n = 0. \tag{19}$$

Indeed, last statement immediately follows from the inequalities

$$\eta q^{\sigma_0 n} - M q^{(\sigma_0+\delta)n} \le x_n \le \eta q^{\sigma_0 n}, \quad n < \frac{1}{\delta}\log_q \frac{\eta}{M}, \quad n \in \mathbb{Z}.$$

We will prove below that

$$\lim_{n\to+\infty} x_n = \eta. \tag{20}$$

Since $x_n \uparrow$, $n \in \mathbb{Z}$, from (18) it immediately follows that $0 < \lambda := \lim\limits_{n\to+\infty} x_n \le \eta$.

Taking the limit of both sides of (1) as $n \to +\infty$, from conditions $a) - d)$ and the limit property of the convolution type discrete operators we get

$$\lambda = g(\lambda), \ \lambda \in (0, \eta].$$

However the last is possible only when $\lambda = \eta$.

Now we prove that the constructed solution x has the following additional properties

$$h - \mathrm{x} \in l_1,$$

where $h = (\dots, \eta, \eta, \dots)^T$ and $\mathrm{x} = (\dots, x_{-1}, x_0, x_1, \dots)^T$; so we will prove that

$$\sum_{n=0}^{\infty} (\eta - x_n) < +\infty. \tag{21}$$

Since $\lim\limits_{n\to\infty} x_n = \eta$ and $x_n \uparrow$, $n \in \mathbb{Z}$, there exists a number $n_0 \in \mathbb{N}$, such that for $n \ge n_0$ we have $x_n \ge \dfrac{\eta}{2}$. Fix a number n_0. Obviously, $\sum\limits_{n=0}^{n_0-1} (\eta - x_n)$ is finite. So we can consider the following quantity

$$\sum_{n=n_0}^{N} (\eta - x_n), \tag{22}$$

where $N > n_0$ is an arbitrary number. In light of (1)–(3), $a) - d)$, from (22) we have

$$\begin{aligned}
\sum_{n=n_0}^{N} (\eta - x_n) &= \sum_{n=n_0}^{N} \left(\eta - \sum_{j=-\infty}^{\infty} a_{n-j} g(x_j) \right) = \sum_{n=n_0}^{N} \sum_{j=-\infty}^{\infty} (\eta - g(x_j))\, a_{n-j} \\
&\le \sum_{n=n_0}^{N} \eta \sum_{j=-\infty}^{0} a_{n-j} + \sum_{n=n_0}^{N} \sum_{j=1}^{\infty} (\eta - g(x_j)) a_{n-j} \\
&= \eta \sum_{n=n_0}^{N} \sum_{i=n}^{\infty} a_i + \sum_{n=n_0}^{N} \sum_{j=1}^{\infty} (\eta - g(x_j)) a_{n-j}
\end{aligned}$$

$$\leq \eta\sum_{i=0}^{\infty} ia_i + \sum_{n=n_0}^{N}\sum_{j=1}^{n}(\eta - g(x_j))a_{n-j} + \sum_{n=n_0}^{N}\sum_{j=n+1}^{\infty}(\eta - g(x_j))a_{n-j}$$
$$\leq c_1 + \sum_{n=n_0}^{N}\sum_{j=1}^{n}(\eta - g(x_j))a_{n-j} + \sum_{n=n_0}^{N}(\eta - g(x_{n+1}))\sum_{j=n+1}^{\infty}a_{n-j}$$
$$\leq c_1 + \sum_{n=n_0}^{N}\sum_{j=1}^{n}(\eta - g(x_j))a_{n-j} + \sum_{n=n_0}^{N}(\eta - g(x_n))\sum_{j=n+1}^{\infty}a_{n-j}$$
$$= c_1 + \sum_{n=n_0}^{N}(\eta - g(x_n))\sum_{i=-\infty}^{-1}a_i + \sum_{n=n_0}^{N}\sum_{j=1}^{n_0-1}(\eta - g(x_j))a_{n-j} + \sum_{n=n_0}^{N}\sum_{j=n_0}^{n}(\eta - g(x_j))a_{n-j}$$
$$\leq c_1 + \eta\sum_{n=n_0}^{\infty}\sum_{j=1}^{n_0-1}a_{n-j} + \sum_{n=n_0}^{N}(\eta - g(x_n))\sum_{i=-\infty}^{-1}a_i + \sum_{n=n_0}^{N}\sum_{j=n_0}^{n}(\eta - g(x_j))a_{n-j}$$
$$= c_1 + c_2 + \sum_{n=n_0}^{N}(\eta - g(x_n))\sum_{i=-\infty}^{-1}a_i + \sum_{j=n_0}^{N}(\eta - g(x_j))\sum_{n=j}^{N}a_{n-j}$$
$$= c + \sum_{n=n_0}^{N}(\eta - g(x_n))\sum_{i=-\infty}^{-1}a_i + \sum_{j=n_0}^{N}(\eta - g(x_j))\sum_{i=0}^{N-j}a_i$$
$$\leq c + \sum_{n=n_0}^{N}(\eta - g(x_n)),$$

where

$$c_1 := \eta\sum_{i=0}^{\infty} ia_i < +\infty, \quad c_2 := \eta\sum_{n=n_0}^{\infty}\sum_{j=1}^{n_0-1}a_{n-j} < +\infty, \quad c = c_1 + c_2.$$

From this it follows that

$$\sum_{n=n_0}^{N}(g(x_n) - x_n) \leq c. \tag{23}$$

Now we consider the line passing through the points $(x_{n_0}, g(x_{n_0}))$ and (η, η)

$$y = \frac{\eta - g(x_{n_0})}{\eta - x_{n_0}}u + \eta\frac{g(x_{n_0}) - x_{n_0}}{\eta - x_{n_0}}.$$

Since $x_{n_0} \leq x_n \leq \eta$ for $n \geq n_0$, then by the convexity of the function g we get

$$g(x_n) \geq \frac{\eta - g(x_{n_0})}{\eta - x_{n_0}}x_n + \eta\frac{g(x_{n_0}) - x_{n_0}}{\eta - x_{n_0}}, \quad n \geq n_0, \quad n \in \mathbb{Z}. \tag{24}$$

From the last inequality it follows that

$$g(x_n) - x_n \geq x_n\left(\frac{\eta - g(x_{n_0})}{\eta - x_{n_0}} - 1\right) + \eta\frac{g(x_{n_0}) - x_{n_0}}{\eta - x_{n_0}} = (\eta - x_n)\left(\frac{g(x_{n_0}) - x_{n_0}}{\eta - x_{n_0}}\right).$$

Since $g(u) > u$ on $(0, \eta)$, then

$$\frac{g(x_{n_0}) - x_{n_0}}{\eta - x_{n_0}} > 0. \tag{25}$$

Taking into account (23) and (25), we come to the following inequalities

$$\frac{g(x_{n_0}) - x_{n_0}}{\eta - x_{n_0}} \sum_{n=n_0}^{N} (\eta - x_n) \le \sum_{n=n_0}^{N} (g(x_n) - x_n) \le c.$$

Now, taking the limit of both sides of this inequality as $N \to +\infty$, we obtain

$$\sum_{n=n_0}^{\infty} (\eta - x_n) < +\infty.$$

So the following theorem holds.

Theorem 1. *Let conditions* (2), (3), $a) - d)$ *and* (6) *are satisfied. Then the system* (1) *has a bounded, nonnegative, nontrivial solution* $\mathrm{x} = (\dots, x_{-1}, x_0, x_1, \dots)^T$, *such that*

- $x_n \uparrow$ *on* $n \in \mathbb{Z}$;
- $\lim\limits_{n\to-\infty} x_n = 0, \quad \lim\limits_{n\to+\infty} x_n = \eta$;
- $\sum\limits_{n=0}^{\infty} (\eta - x_n) < +\infty.$

Uniqueness of Solution. We prove uniqueness of the solution in the following class:

$$\mathfrak{M} = \{\mathrm{x} = (\dots, x_{-1}, x_0, x_1, \dots)^T : \ \mathcal{L}_n \le x_n \le x_n^{(0)}, \ n \in \mathbb{Z}\}.$$

Assume to the contrary that system (1) has two different solutions

$$\mathrm{x} = (\dots, x_{-1}, x_0, x_1, \dots)^T, \ \ \tilde{\mathrm{x}} = (\dots, \tilde{x}_{-1}, \tilde{x}_0, \tilde{x}_1, \dots)^T \in \mathfrak{M}.$$

Then it is easy to see that the sequence

$$q^{-(\sigma_0+\delta)n} |x_n - \tilde{x}_n|, \quad n \in \mathbb{Z},$$

is bounded.

Since $\mathcal{L}_n = 0$ when $n \ge \frac{1}{\delta} \log_q \frac{\eta}{M}$, $n \in \mathbb{Z}$, by virtue of the definition of $x_n^{(0)}$ for $n \ge \frac{1}{\delta} \log_q \frac{\eta}{M}$, $n \in \mathbb{Z}$, it holds

$$q^{-(\sigma_0+\delta)n} |x_n - \tilde{x}_n| \le 2\eta q^{-(\sigma_0+\delta)n} \le \left(\frac{M}{\eta}\right)^{1+\frac{\sigma_0}{\delta}}.$$

Now let $n < \frac{1}{\delta} \log_q \frac{\eta}{M}$, $n \in \mathbb{Z}$. Then due to $\mathrm{x}, \ \tilde{\mathrm{x}} \in \mathfrak{M}$, we obtain

$$-M q^{(\sigma_0+\delta)n} \le x_n - \tilde{x}_n \le M q^{(\sigma_0+\delta)n}$$

or

$$q^{-(\sigma_0+\delta)n} |x_n - \tilde{x}_n| \le M.$$

Thus

$$\alpha := \sup_{n\in\mathbb{Z}} q^{-(\delta+\sigma_0)n} |x_n - \tilde{x}_n| < +\infty.$$

Since the function g is convex on $[0,\eta]$, we obtain

$$|g(x_n)-g(\tilde{x}_n)|\leq g'(0)|x_n-\tilde{x}_n|, \quad n\in\mathbb{Z}. \tag{26}$$

Taking into account (26), from (1) we get

$$|x_n-\tilde{x}_n|\leq \sum_{j=-\infty}^{\infty} a_{n-j}|g(x_j)-g(\tilde{x}_j)|\leq g'(0)\sum_{j=-\infty}^{\infty} a_{n-j}|x_j-\tilde{x}_j|q^{-(\sigma_0+\delta)j}q^{(\sigma_0+\delta)j}$$
$$\leq g'(0)\alpha\sum_{j=-\infty}^{\infty} a_{n-j}q^{(\sigma_0+\delta)j}=g'(0)\alpha q^{(\sigma_0+\delta)n}\sum_{i=-\infty}^{\infty} a_i q^{-(\sigma_0+\delta)i}$$
$$=\alpha q^{(\sigma_0+\delta)n}L(\sigma_0+\delta),$$

from which it follows that

$$q^{-(\sigma_0+\delta)n}|x_n-\tilde{x}_n|\leq \alpha L(\sigma_0+\delta), \quad n\in\mathbb{Z}. \tag{27}$$

From (27) we conclude that

$$\alpha\leq \alpha L(\delta+\sigma_0). \tag{28}$$

From (8) we have that $L(\delta+\sigma_0)<1$ if $\delta\in(0,\min\{\lambda_0-\sigma_0,\sigma_0\varepsilon\})$, and then from (28) it follows that $\alpha=0$. Therefore $x_n=\tilde{x}_n$, $n\in\mathbb{Z}\ \Rightarrow\ \mathrm{x}=\tilde{\mathrm{x}}$.

Thus the following theorem holds.

Theorem 2. *Under the conditions of Theorem 1, system (1) has a unique solution in* $\mathfrak{M}$.

Let us list some examples of nonlinear function g:

I) $g(u)=\gamma(1-e^{-u}),\quad \gamma>1,\quad u\geq 0;$

II) $g(u)=\gamma(u-u^{1+\varepsilon}),\quad u\geq 0,\quad \gamma>1,\quad \varepsilon>0.$

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Մ. Հ. ԱՎԵՏԻՍՅԱՆ

ՄԻ ՈՉ ԳԾԱՅԻՆ ԴԻՍԿՐԵՏ ՀԱՄԱԿԱՐԳԻ ԼՈՒԾԵԼԻՈՒԹՅՈՒՆԸ ՀԱՄԱՃԱՐԱԿԻ ՏԱՐԱԾՄԱՆ ՏԵՍՈՒԹՅԱՆ ՄԵՋ

Տվյալ աշխատանքում հետազոտվում է Տյոպլիցյան մատրիցով հատուկ դասի ոչ գծային անվերջ հանրահաշվական հավասարումների համակարգ: Դիտարկվող համակարգը ծագում է համաճարակի տարածաժամանակային տարածման մաթեմատիկական տեսության մեջ: Ապացուցվում են սահմանափակ հաջորդականությունների դասում լուծման գոյության և միակության թեորեմներ: Հետազոտվում է նաև կառուցվող լուծման ասիմպտոտիկ վարքն անվերջությունում: Աշխատանքի վերջում բերվում են կիրառական նշանակություն ունեցող օրինակներ:

М. О. АВЕТИСЯН

О РАЗРЕШИМОСТИ ОДНОЙ НЕЛИНЕЙНОЙ ДИСКРЕТНОЙ СИСТЕМЫ В ТЕОРИИ РАСПРОСТРАНЕНИЯ ЭПИДЕМИИ

В данной работе исследуется специальный класс системы нелинейных бесконечных алгебраических уравнений с матрицами Теплица. Указанная система возникает в математической теории пространственно-временного распространения эпидемии. Доказываются теоремы существования и единственности решения в пространстве ограниченных последовательностей. Исследуется также асимптотическое поведение построенных решений в бесконечности. В конце приводятся специальные прикладные примеры.