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Mathematics

THE SOLVABILITY OF AN INFINITE SYSTEM OF NONLINEAR ALGEBRAIC EQUATIONS WITH TOEPLITZ MATRIX

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The work is focused on studying of the existence, uniqueness, and various qualitative properties of the constructive solution of an infinite system of algebraic equations with a concave nonlinearity property, which are generated by Toeplitz matrices. In addition to its independent mathematical interest, such systems have a significant application in several branches of mathematical physics and mathematical biology. Those particularly appear in discrete problems within radiative transfer theory, kinetic theory of gases, dynamic theory of p-adic strings, and the mathematical theory of epidemic propagation. We establish the existence of a positive solution for the system in the class of bounded sequences, as well as provide an iterative method to approximate to the solution. We also study the asymptotic behavior of the solution at infinity and the uniqueness of the nontrivial solution with non-negative elements in the class of bounded sequences. The last section of the paper provides examples of applications of the corresponding Toeplitz matrix and the function that describes the nonlinearity.

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Introduction. The work is focused on the following infinite system of nonlinear algebraic equations:

$$x_i = \sum_{j=0}^{\infty} a_{i-j} G(x_j), \quad i = 0, 1, 2, \ldots, \tag{1}$$

with respect to an infinite vector $\mathrm{x} = (x_0, x_1, \ldots, x_n, \ldots)^T \in m$ with non-negative coordinates, where T is the transposition sign and m is the class of bounded sequences.

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It is assumed that the elements of Toeplitz matrix $A = (a_{i-j})_{i,j=0}^{\infty}$ satisfy the following conditions:

1) $a_m > 0,\ \forall m \in \mathbb{Z} = \{0, \pm 1, \pm 2, \dots\},\ \sum_{m=-\infty}^{\infty} a_m = 1;$

2) $\sum_{m=-\infty}^{\infty} |m| a_m < +\infty.$

The function G, which describes the nonlinearity, satisfies the following properties:

a) $G \in C(\mathbb{R}^+)$, and G is monotonically increasing on $\mathbb{R}^+ = [0, +\infty)$;

b) $G(0) = 0$, and there exists $\eta > 0$ such that $G(\eta) = \eta$;

c) $y = G(u)$ is strictly concave on $\mathbb{R}^+$;

d) there exists $\varphi : [0, 1] \to [0, 1]$ continuous, monotonous, concave function such that $\varphi(0) = 0,\ \varphi(1) = 1$ and $G(\sigma u) \geq \varphi(\sigma) G(u)$ for any $\sigma \in [0, 1], u \in [0, \eta]$.

Beyond its theoretical interest, the study of system (1) is crucial in determining the solvability of various discrete model problems in physics and biology. In particular, certain equations like (1) are used in the radiative transfer theory in an inhomogeneous mediums, kinetic theory of gases (within the framework of the model of Bhatnagar–Gross–Kruk), in the mathematical theory of epidemic distribution (within modified discrete models of Diekmann–Kaper and Atkinson–Reuter) and dynamic theory of p-adic strings (see [1–5]).

It is noteworthy that when $G(u) = u$, the system (1) turns into Wiener–Hopf type discrete equations, a subject that is extensively studied in numerous works (see for instance [6–9]). In the case when $G(u) = u^{\alpha}$, $0 < \alpha < 1$, and $\nu(A) = \sum_{m=-\infty}^{\infty} m a_m < 0$, the system (1) has been considered in [10]. It was proved that a non-negative solution exists for system (1) in the class of bounded sequences. Later, that result was generalized in the case when $G(u)$ satisfies a)–c) and $\nu(A) < 0$ (see [11]). Note that papers [10] and [11] did not consider the uniqueness problem and the provided proofs of existence theorems are not constructive.

In this work, under conditions 1)–2) and a)–d), it becomes possible to establish the existence of a non-negative solution of (1) in the class of bounded sequences. Besides, it provides a successive approximations converging to the solution with a geometric progression rate. Additionally, the work establishes the convergence of the positive series $\sum_{n=0}^{\infty} (G(x_n) - x_n)$. Utilizing this fact alongside Jensen's inequality and some a'priori estimates for concave functions, we successfully demonstrate the uniqueness of the solution for system (1) within the class of nontrivial bounded sequences with non-negative elements when $a_{-n} = a_n, n = 1, 2, \dots$ At the end of this paper, examples of matrix A and nonlinearity G are provided.

The Existence of the Nontrivial Solution. In this section, we are going to prove the existence of a solution of the system (1) under conditions 1) and a)–d).

Theorem 1. *Under conditions 1) and a)–d), infinite system* (1) *of nonlinear equations has a positive solution* $\mathrm{x}^* = (x_0^*, x_1^*, \ldots, x_n^*, \ldots)$ *in the class* m*, moreover, there exists* $\lim_{i\to\infty} x_i^* = \eta$.

Additionally, successive approximations

$$\begin{cases} x_i^{*(k+1)} = \sum_{j=0}^{\infty} a_{i-j} G(x_j^{*(k)}), \\ x_i^{*(0)} = \eta, \end{cases} \qquad i = 0,1,2\ldots, \quad k = 0,1,2,\ldots, \tag{2}$$

provided for system (1) *converge to the system solution in a geometric progression rate.*

Proof. Consider the successive approximations (2). It is easy to verify by induction that for any $i = 0,1,2,\ldots$

$$x_i^{*(k)} \text{ is decreasing with respect to } k \tag{3}$$

and

$$x_i^{*(k)} > 0, \quad k = 0,1,2,\ldots \tag{4}$$

Denote $\sigma_0 := \sum_{s=-\infty}^{0} a_s$. Condition 1) immediately implies that $\sigma_0 \in (0,1)$.

From (2)–(4) we get the following simple inequality $\sigma_0 x_i^{*(0)} \le x_i^{*(1)} \le x_i^{*(0)}$, $i = 0,1,2,\ldots$ Taking into account the monotonicity of function G and 1), from the preceding inequality and from (2) we come to the following inequality:

$$\sum_{j=0}^{\infty} a_{i-j} G(\sigma_0 x_j^{*(0)}) \le \sum_{j=0}^{\infty} a_{i-j} G(x_j^{*(1)}) \le \sum_{j=0}^{\infty} a_{i-j} G(x_j^{*(0)}), \quad i = 0,1,2,\ldots \tag{5}$$

From (2), (5) and condition d), the following double inequality is immediately derived

$$\varphi(\sigma_0) x_i^{*(1)} \le x_i^{*(2)} \le x_i^{*(1)}, \quad i = 0,1,2,\ldots \tag{6}$$

Once again considering the monotonicity of function G and the conditions 1) and d), from the inequality (6) we obtain the following chain of inequalities:

$$\varphi(\varphi(\sigma_0)) x_i^{*(2)} \le x_i^{*(3)} \le x_i^{*(2)}.$$

Continuing this process, it is straightforward to confirm that at the k-th step of induction, we reach the following inequalities:

$$F_k(\sigma_0) x_i^{*(k)} \le x_i^{*(k+1)} \le x_i^{*(k)}, \; i = 0,1,2,\ldots, \; k = 1,2,\ldots, \tag{7}$$

where $F_k(\sigma) := \underbrace{\varphi(\varphi(\ldots\varphi(\sigma)\ldots))}_{k}$.

Denote by $l := \dfrac{1-\varphi\left(\frac{\sigma_0}{2}\right)}{1-\frac{\sigma_0}{2}}$. It follows from the properties of function φ that $l \in (0,1)$. It is easy to show that $\varphi(\sigma_0) \ge l\sigma_0 + 1 - l$, which, in its turn, implies that $\varphi(\varphi(\sigma_0)) \geq \varphi(l\sigma_0+1-l) \geq l(l\sigma_0+1-l)+1-l = l^2\sigma_0+1-l^2$. Once more, employing inductive reasoning, at the k-th step we obtain

$$F_k(\sigma_0) \geq l^k\sigma_0+1-l^k, \quad k=1,2,\dots \tag{8}$$

From (3), (7) and (8) we get that

$$0 \leq x_i^{*(k)} - x_i^{*(k+1)} \leq \eta(1-\sigma_0)l^k,\ i=0,1,2,\dots,\ k=0,1,2,\dots \tag{9}$$

We conclude from (3), (4) and (9) that the sequence of infinite vectors $\mathrm{x}^{*(k)} := (x_0^{*(k)}, x_1^{*(k)}, \dots, x_n^{*(k)}, \dots)^T$, $k=0,1,2,\dots$, with non-negative coordinates converges to the vector $\mathrm{x}^* = (x_0^*, x_1^*, \dots, x_n^*, \dots)^T \in m$, i.e. $\lim\limits_{k\to\infty} x_i^{*(k)} = x_i^*$, $i=0,1,2,\dots$, moreover,

$$0 \leq x_i^* \leq \eta, \quad i=0,1,2,\dots \tag{10}$$

Taking into account the continuity of the function G and the evident double inequality $\sum\limits_{j=0}^{\infty} a_{i-j}G(x_j^{*(k)}) \leq \eta \sum\limits_{j=0}^{\infty} a_{i-j} \leq \eta$ for $k=0,1,2,\dots$ and $i=0,1,2,\dots$, we conclude that the infinite vector $\mathrm{x}^* = (x_0^*, x_1^*, \dots, x_n^*, \dots)^T$ is a solution of the system (1).

Now, we write the inequality (9) for $k+1, k+2, \dots, k+t$:

$$\begin{cases} 0 \leq x_i^{*(k+1)} - x_i^{*(k+2)} \leq \eta(1-\sigma_0)l^{k+1}, \\ 0 \leq x_i^{*(k+2)} - x_i^{*(k+3)} \leq \eta(1-\sigma_0)l^{k+2}, \\ \vdots \\ 0 \leq x_i^{*(k+t-1)} - x_i^{*(k+t)} \leq \eta(1-\sigma_0)l^{k+t}, \end{cases} \qquad i=0,1,2,\dots$$

By adding the left and right hand sides of obtained inequalities to the inequality (9), we get $0 \leq x_i^{*(k)} - x_i^{*(k+t)} \leq \dfrac{\eta(1-\sigma_0)l^k}{1-l}$, $i=0,1,2,\dots$ In the last inequality by fixing k and passing to the limit as $t\to\infty$, we obtain the following uniform estimation:

$$0 \leq x_i^{*(k)} - x_i^* \leq \frac{\eta(1-\sigma_0)l^k}{1-l},\ i=0,1,2,\dots, \quad k=1,2,\dots \tag{11}$$

We now ensure that there exists $\lim\limits_{i\to\infty} x_i^* = \eta$. To achieve this, we first note that it is straightforward to verify by induction with respect to k that

$$\lim_{i\to\infty} x_i^{*(k)} = \eta,\ \ k=0,1,2\dots \tag{12}$$

Then, we show that $x_{i+1}^* \geq x_i^*$. Indeed, by initially rewriting the successive approximations (2) as

$$\begin{cases} x_i^{*(k+1)} = \sum\limits_{s=-\infty}^{i} a_s G(x_{i-s}^{*(k)}), \\ x_i^{*(0)} = \eta, \end{cases} \qquad i=0,1,2,\dots,$$

and using induction with respect to k with monotonicity of function G, it is easy to verify that $x_{i+1}^{*(k)} \geq x_i^{*(k)}$ for any $k \in \mathbb{Z}^+ := \{0,1,2,\dots\}$. Passing to the limit as $k\to\infty$ we get the inequality $x_{i+1}^* \geq x_i^*$. Since $0 \leq x_i^* \leq \eta$, then there exists

$\lim\limits_{i\to\infty} x_i^* =: \lambda \leq \eta$. Passing to the limit as $i \to \infty$ in inequality (11), we obtain that $0 \leq \eta - \lambda \leq \dfrac{\eta(1-\sigma_0)l^k}{1-l}$, for any $k = 1,2,\ldots$ Since $0 < l < 1$, passing to the limit as $k \to \infty$ yields $\lambda = \eta$. □

Some Properties of a Non-negative Nontrivial Solution of System (1). The following theorem holds:

Theorem 2. *Let conditions 1) and a)–c) hold. Then, for any* $\mathrm{x} = (x_0, x_1, \ldots, x_n, \ldots)^T$ *nontrivial, non-negative bounded solution of system* (1) *the following double inequality holds:*

$$0 < x_i < \eta, \quad i = 0,1,2,\ldots \tag{13}$$

Moreover, if the condition 2) holds, then

$$\sum_{i=0}^{\infty}(G(x_i) - x_i) < +\infty. \tag{14}$$

Proof. Since there exists $i_0 \in \mathbb{Z}^+$ such that $x_{i_0} > 0$, then from conditions a), b) and 1), the inequality $x_i \geq a_{i-i_0}G(x_{i_0}) > 0,\ i = 0,1,2,\ldots,$ is immediately obtained.

Now we prove the inequality $x_i < \eta,\ i = 0,1,2,\ldots$ To achieve that, we initially prove that $x_i \leq \eta,\ i = 0,1,2,\ldots$ Denote by $c := \sup\limits_{i\in\mathbb{Z}^+} x_i$. Taking into account conditions 1) and a), b), from system (1) we get that $x_i \leq G(c)$. This immediately implies that $c \leq G(c)$. Since $c > 0$ and the function $\dfrac{G(u)}{u}$ is monotonically decreasing on $(0, +\infty)$, then from the last inequality and the condition b) it follows that $c \leq \eta$. Notice that $x_i \not\equiv \eta$, hence there exists $i^* \in \mathbb{Z}^+$ such that $x_{i^*} < \eta$. Therefore, considering conditions 1) and a), it is obtained from (1) that $x_i < \eta,\ i = 0,1,2,\ldots$

We now proceed to prove the second part of the theorem. Initially employing (13) and condition 1), we have

$$0 < \eta - x_i = \eta \sum_{j=-\infty}^{\infty} a_{i-j} - \sum_{j=0}^{\infty} a_{i-j}G(x_j) = \eta \sum_{j=-\infty}^{-1} a_{i-j} + \sum_{j=0}^{\infty} a_{i-j}(\eta - G(x_j)).$$

Denote by $I_i := \eta \sum\limits_{j=-\infty}^{-1} a_{i-j},\ i = 0,1,2,\ldots$ Let us prove, that

$$\sum_{i=0}^{\infty} I_i < +\infty. \tag{15}$$

Under the conditions 1) and 2) we have that for any positive integer N

$$\sum_{i=0}^{N} I_i = \eta \sum_{i=0}^{N} \sum_{s=i+1}^{\infty} a_s = \eta \sum_{i=0}^{N} \sum_{s=i+1}^{N+1} a_s + \eta \sum_{i=0}^{N} \sum_{s=N+2}^{\infty} a_s$$
$$= \eta \sum_{s=1}^{N+1} s a_s + \eta \sum_{s=N+2}^{\infty} a_s(N+1) \leq \eta \sum_{s=1}^{\infty} s a_s < +\infty.$$

Passing to the limit as $N \to \infty$ we come to (15). Now, we denote by $c_1 := \eta \sum_{s=1}^{\infty} s a_s$ and $c_2 := \eta \sum_{s=0}^{\infty} (s+1) a_{-s} < +\infty$, and let N_1 be some positive integer. Once again considering conditions 1), 2), a) along with (15), we deduce

$$\begin{aligned}
\sum_{i=0}^{N_1} (\eta - x_i) &\le c_1 + \sum_{i=0}^{N_1} \sum_{j=0}^{\infty} a_{i-j}(\eta - G(x_j)) \\
&= c_1 + \sum_{i=0}^{N_1} \sum_{j=0}^{N_1} a_{i-j}(\eta - G(x_j)) + \sum_{i=0}^{N_1} \sum_{j=N_1+1}^{\infty} a_{i-j}(\eta - G(x_j)) \\
&\le c_1 + \sum_{j=0}^{N_1} (\eta - G(x_j)) \sum_{i=0}^{N_1} a_{i-j} + \eta \sum_{i=0}^{N_1} \sum_{s=-\infty}^{i-N_1-1} a_s \\
&\le c_1 + \sum_{j=0}^{N_1} (\eta - G(x_j)) + \eta \sum_{i=0}^{N_1} \sum_{s=N_1-i}^{\infty} a_{-s} \\
&= c_1 + \sum_{j=0}^{N_1} (\eta - G(x_j)) + \eta \sum_{k=0}^{N_1} \sum_{s=k}^{\infty} a_{-s} \\
&\le c_1 + \sum_{j=0}^{N_1} (\eta - G(x_j)) + c_2,
\end{aligned}$$

from which we get

$$\sum_{i=0}^{N_1} (G(x_i) - x_i) \le c_1 + c_2 < +\infty.$$

Passing to the limit as $N_1 \to +\infty$, we come to (14). □

Remark. *Note that Theorem 2 and existence Theorem 1 imply*

$$\sum_{i=0}^{\infty} (\eta - x_i^*) < +\infty. \tag{16}$$

Indeed, since $\lim_{i\to\infty} x_i^* = \eta$, *then there exists* $i_0 \in \mathbb{Z}^+$ *such that* $x_i^* \ge \frac{\eta}{2}$ *for any* $i > i_0$. *Considering conditions a)–c) alongside inequality (13), we can establish*

$$\frac{\eta - G(x_i^*)}{\eta - x_i^*} \le \frac{\eta - G\left(\frac{\eta}{2}\right)}{\frac{\eta}{2}} =: \alpha \in (0, 1),$$

from which $G(x_i^*) - x_i^* \ge (1-\alpha)(\eta - x_i^*)$. *From the resulted inequality and due to* $\sum_{i=0}^{\infty} (G(x_i^*) - x_i^*) < +\infty$, *(16) is derived.*

The Uniqueness of the Solution. Examples.

Theorem 3. *Under conditions of Theorem 1, if* $a_{-i} = a_i$, $i = 1, 2 \ldots,$ *holds alongside condition 2), then the system (1) does not have more than one solution within the class of non-negative, nontrivial bounded sequences.*

Proof. For the sake of contradiction suppose that the system (1) apart from the solution $\mathrm{x}^* = (x_0^*, x_1^*, \ldots, x_n^*, \ldots)$ (constructed by the successive approximations (2)) also has another solution $\mathrm{x} = (x_0, x_1, \ldots, x_n, \ldots)$ within the class of non-negative, nontrivial, bounded sequences. By using Theorem 2 and applying induction on k, it is easy to check that $x_i \le x_i^{*(k)}$ for any $i = 0,1,2,\ldots,\ k = 0,1,2,\ldots$ Passing to the limit as $k \to \infty$, we have

$$x_i \le x_i^*, \quad i = 0,1,2,\ldots \tag{17}$$

Hence

$$0 \le x_i^* - x_i = \sum_{j=0}^{\infty} a_{i-j}(G(x_j^*) - G(x_j)), \quad i = 0,1,2,\ldots \tag{18}$$

Denote by Q the reverse function of G on $\mathbb{R}^+$. Considering (14) and the condition $a_{-i} = a_i$, we have

$$\begin{aligned}\sum_{i=0}^{\infty}(G(x_i)-x_i)(x_i^*-x_i) &= \sum_{i=0}^{\infty}(G(x_i)-x_i)\sum_{j=0}^{\infty}a_{i-j}(G(x_j^*)-G(x_j))\\ &= \sum_{j=0}^{\infty}(G(x_j^*)-G(x_j))\sum_{i=0}^{\infty}a_{i-j}(G(x_i)-x_i)\\ &= \sum_{j=0}^{\infty}(G(x_j^*)-G(x_j))\left(\sum_{i=0}^{\infty}a_{j-i}G(x_i)-\sum_{i=0}^{\infty}a_{j-i}x_i\right)\\ &= \sum_{j=0}^{\infty}(G(x_j^*)-G(x_j))\left(x_j-\sum_{i=0}^{\infty}a_{j-i}Q(G(x_i))\right).\end{aligned}$$

Taking into account the easily verifiable inequality $vQ(u) \ge Q(vu),\ v \in [0,1]$, $u \in \mathbb{R}^+$ with Jensen's inequality, we have

$$\begin{aligned}\sum_{i=0}^{\infty}a_{j-i}Q(G(x_i)) &\ge \sum_{i=0}^{\infty}a_{j-i}Q\left(\frac{\sum_{i=0}^{\infty}a_{j-i}G(x_i)}{\sum_{i=0}^{\infty}a_{j-i}}\right)\\ &\ge Q\left(\sum_{i=0}^{\infty}a_{j-i}G(x_i)\right) = Q(x_j).\end{aligned} \tag{19}$$

Thus, by using (19), we reach the following inequality

$$\sum_{i=0}^{\infty}(G(x_i)-x_i)(x_i^*-x_i) \le \sum_{j=0}^{\infty}(G(x_j^*)-G(x_j))(x_j-Q(x_j)). \tag{20}$$

Define the following set of indices

$$\mathcal{P} := \{i \in \mathbb{Z}^+ : x_i^* > x_i\}.$$

According to our assumption $\mathcal{P} \ne \varnothing$ and $\mathcal{P}^c := \mathbb{Z}^+ \setminus \mathcal{P} = \{i \in \mathbb{Z}^+ : x_i^* = x_i\}$. From the definition of the set $\mathcal{P}$ and inequality (13), one can see that inequality (20) can be rewritten as

$$\sum_{i\in\mathcal{P}}(x_i^*-x_i)(x_i-Q(x_i))\left(\frac{G(x_i)-x_i}{x_i-Q(x_i)}-\frac{G(x_i^*)-G(x_i)}{x_i^*-x_i}\right) \le 0. \tag{21}$$

On the other hand, when $i \in \mathcal{P}$, the following inequalities hold:

$$x_i^* > x_i > Q(x_i) > 0,$$
$$\frac{G(x_i) - x_i}{x_i - Q(x_i)} = \frac{G(x_i) - G(Q(x_i))}{x_i - Q(x_i)} > \frac{G(x_i^*) - G(x_i)}{x_i^* - x_i}.$$

These inequalities contradict with (21), hence $\mathcal{P} = \varnothing$, that is $x_i^* = x_i$, $i = 0, 1, 2, \ldots$ □

At the end of this work, we provide a few examples of theoretical nature of Toeplitz matrix A and the nonlinearity G.

Examples of Toeplitz matrix A:

1*) $\begin{cases} a_n = \dfrac{45}{2\pi^4} \cdot \dfrac{1}{n^4}, \; n = \pm 1, \pm 2, \ldots, \\ a_0 = \dfrac{1}{2}; \end{cases}$

2*) $a_n = \dfrac{1-q}{1+q} \cdot q^{|n|}, \quad 0 < q < 1;$

3*) $\begin{cases} a_n = \dfrac{1}{2}\left(\dfrac{45}{2\pi^4} \cdot \dfrac{1}{n^4} + \dfrac{1-q}{1+q} \cdot q^{|n|}\right), \; n = \pm 1, \pm 2, \ldots, \\ a_0 = \dfrac{1}{2}\left(\dfrac{1}{2} + \dfrac{1-q}{1+q}\right), \end{cases} \qquad 0 < q < 1.$

Examples of function G:

a^*) $G(u) = u^\alpha$, $\varphi(\sigma) = \sigma^\alpha$, $\quad 0 < \alpha < 1$;

b^*) $G(u) = \dfrac{u^\alpha + u^\beta}{2}, \varphi(\sigma) = \sigma^{\frac{\alpha+\beta}{2}}, \quad 0 < \alpha, \beta < 1$;

c^*) $G(u) = \gamma(1 - e^{-u^\alpha})$, $\varphi(\sigma) = \sigma^\alpha$, $\quad \gamma > 1$, $0 < \alpha < 1$.

It should be noted that provided examples 2^*), a^*) and c^*) besides of theoretical significance, are also interesting in the context of discrete problems in the dynamic theory of p-adic strings and the mathematical theory of spatial-temporal spread of epidemic diseases.

Conclusion. In this work, sufficient conditions for the existence of a positive solution to system (1) in the space of bounded sequences are obtained. The asymptotic behavior of the constructed solution in the class of non-negative, non-trivial and bounded sequences is studied. At the end, specific examples of the matrix A and nonlinearity G are given that satisfy all the conditions of the proved theorems.

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ՏՅՈՊԼԻՑՅԱՆ ՄԱՏՐԻՑՈՎ ՄԻ ՈՉ ԳԾԱՅԻՆ ՀԱՆՐԱՀԱՇՎԱԿԱՆ ՀԱՎԱՍԱՐՈՒՄՆԵՐԻ ԱՆՎԵՐՋ ՀԱՄԱԿԱՐԳԻ ԼՈՒԾԵԼԻՈՒԹՅՈՒՆԸ

Աշխատանքը նվիրված է տյոպլիցյան մատրիցներով ծնվող զոգավոր ոչ գծայնությամբ հանրահաշվական հավասարումների անվերջ համակարգի կոնստրուկտիվ լուծման գոյության, միակության և լուծման որոշ որակական հատկությունների ուսումնասիրման հարցերին:

Դիտարկվող համակարգը, բացի ինքնուրույն մաթեմատիկական հետաքրքրությունից, ունի նաև կիրառական կարևոր նշանակություն մաթեմատիկական ֆիզիկայի և մաթեմատիկական կենսաբանության մի շարք ճյուղերում: Մասնավորապես, նշված տիպի համակարգեր ծագում են ճառագայթման տեղափոխման

տեսությունում, գազերի կինետիկ տեսությունում, p-ադիկ լարերի դինամիկ տեսության և համաճարակի տարածման մաթեմատիկական տեսության դիսկրետ խնդիրներում:

Աշխատանքում ապացուցվում է դիտարկվող համակարգի դրական լուծման գոյությունը սահմանափակ հաջորդականությունների դասում: Առաջարկվում է այդ համակարգի մոտավոր լուծման կառուցման մեթոդ: Հետազոտվում է լուծման ասիմպտոտիկ վարքն անվերջությունում: Հաջողվում է նաև ապացուցել ոչ տրիվիալ լուծման միակությունը ոչ բացասական էլեմենտներով սահմանափակ հաջորդականություննեի դասում:

Աշխատանքի վերջում բերվում են համապատասխան տյոպլիցյան մատրիցի և ոչ գծայնությունը նկարագրող ֆունկցիայի կիրառական բնույթի օրինակներ:

Х. А. ХАЧАТРЯН, В. Г. ДИЛАНЯН

О РАЗРЕШИМОСТИ ОДНОЙ НЕЛИНЕЙНОЙ СИСТЕМЫ БЕСКОНЕЧНЫХ АЛГЕБРАИЧЕСКИХ УРАВНЕНИЙ С МАТРИЦАМИ ТИПА ТЕПЛИЦА

Работа посвящена вопросам конструктивной разрешимости, единственности и изучению некоторых качественных свойств решения для одной бесконечной системы алгебраических уравнений с вогнутой нелинейностью и матрицами Теплица. Рассматриваемая система, кроме самостоятельного математического интереса, имеет важный прикладной интерес в различных отраслях математической физики и математической биологии. В частности такие системы возникают в теории переноса излучения, в кинетической теории газов, в динамической теории p-адических струн и в дискретных задачах математической теории распространения эпидемии.

В работе доказывается существование положительного решения для этой системы в пространстве ограниченных последовательностей. Предлагается метод построения приближенного решения данной системы. Исследуется асимптотическое поведение построенного решения. Удается также доказать единственность нетривиального решения в классе ограниченных последовательностей, имеющих неотрицательные элементы.

В конце работы приводятся примеры прикладного характера для соответствующей матрицы Теплица и нелинейности.