PROCEEDINGS OF THE YEREVAN STATE UNIVERSITY

Physical and Mathematical Sciences 2024, **58**(1), p. 13–21

Mechanics

ON OPTIMAL CONTROL OF THERMOELASTIC VIBRATIONS OF A PLATE-STRIP

S. H. JILAVYAN *, E. R. GRIGORYAN **

Chair of Mechanics, YSU, Armenia

The problem of optimal control of elastic vibrations of an isotropic plate-strip under the influence of temperature and force fields is studied. The function of changing the external load on the plane of the plate is represented as a control function. Optimal control is also carried out by the distribution function of the temperature of the external field over the plate. The well-known classical hypotheses of thermo-elastic bending of the plate are accepted. The equations of transverse vibrations of the plate and heat conduction in the plate are solved under the boundary conditions of heat transfer and the stress state on the planes of the plate. The method of Fourier series, the method of representing moment relations, the well-known method of minimizing the functional are used.

https://doi.org/10.46991/PYSU:A.2024.58.1.013

MSC2010: Primary: 35Q93; Secondary: 74F05.

Keywords: vibrations, thermal conductivity, optimal control, thermo-elasticity.

Introduction. The development of control theory and optimal control is driven by important practical tasks in the fields of engineering and technology.The methods of control theory, mathematical physics, and continuum mechanics, taking into account certain peculiarities, make it possible to generalize control theory for solutions to applied technological problems of heat conduction, heat transfer, and thermoelasticity for structural elements of deformable solid bodies. It is widely accepted that these problems are addressed through control methods applied to systems with distributed parameters. The issues of optimal control of temperature fields, optimization of heating, and optimal design of elastic bodies, including plates, are the subjects of studies in [1–6]. Control of thermoelastic processes in structural elements is one of the important and modern branches of deformable solid body mechanics. Special attention is given to the physical-mathematical modeling of mechanical and technical problems

* E-mail: samjilavyan@ysu.am
** E-mail: edmon.grigoryan@ysu.am

that arise in various engineering sectors when designing structures that operate at elevated temperature levels.

Problem Statement. An isotropic rectangular plate-strip with a thickness of 2 h and width a is referred to a rectangular coordinate system x_1, y_1, z_1, $z_1 \in [-h;h]$, $x_1 \in [0;a], y_1 \in (-\infty;\infty)$. The plate is under the influence of a temperature field and the temperature field in the plate $T_*(x_1,z_1,t_1)$, t_1 is time parameter. The interaction between an elastic plate and the surrounding medium is described by well-established laws of thermo-mechanics. Heat exchange occurs between the surfaces of the plate at $z_1 = \pm h$ and the external environment [3–5]

$$\lambda_q \frac{\partial T_*}{\partial z_1} \pm \lambda_c \left(T_* - T_c^{\pm}\right) = 0, \quad z_1 = \pm h, \tag{1}$$

where λ_q is thermal conductivity coefficient; λ_c is heat transfer coefficient of the plate surfaces; $T_c^{\pm}(x_1,t_1)$ is the temperature at $z_1 = \pm h$.

The equations of transverse vibrations of the plate-strip in a temperature field have the form

$$\frac{\partial^2 M_{x_1}}{\partial x_1^2} - 2\rho h \frac{\partial^2 W_1}{\partial t_1^2} = 0. \tag{2}$$

Here, classical hypotheses are adopted for the calculation of isotropic, thin plates $\left(\frac{h^2}{a^2} \ll 1\right)$, $M_{x_1}(x_1,t_1) = -\frac{2}{3}h^3 \frac{E}{1-\nu^2} \cdot \frac{\partial^2 W_1}{\partial x_1^2} - \frac{2}{3}h^2 \frac{E}{1-\nu} \alpha_T T_1$ is bending moment, W_1 is deflection function, $T_1(x_1,t_1) = \frac{3}{2h^2} \int\limits_{-h}^{h} z_1 T_* dz_1$ is the integral characteristic of the temperature field of a thin plate, E, ν, ρ are Young's modulus, Poisson's ratio of the plate material, and the density of the plate material, respectively, α_T is the coefficient of linear thermal expansion. It is known that for a given plate-strip, the functions characterizing thermoelastic vibrations, $W_1(x_1,t_1), T_1(x_1,t_1)$ represent functions of displacement and temperature characterizing thermoelastic vibration. The long sides of the plate $x_1 = 0, x_1 = a$ are hinged

$$W_1 = 0, \quad M_{x_1} = 0, \quad x_1 = 0, \quad x_1 = a. \tag{3}$$

The real process of thermoelastic deformation of a body is, strictly speaking, non-uniform and irreversible. The mechanical energy of the oscillating body (plate) diminishes over time, gradually decreasing from its initial value to the minimum achievable level, ultimately leading to the establishment of an equilibrium state. The change in the accumulated energy within the system is expressed as the rate of mechanical energy dissipation per unit of time.The determination of the temperature field in a thin-walled structural element, such as a plate, is carried out after reducing the spatial heat conduction problem, considering the dissipation of mechanical energy, to a two-dimensional problem. The differential equation of heat conduction concerning the integral characteristic of the temperature function and the displacement function in the plate-strip has the following form:

$$h^2 \cdot \frac{\partial^2 T_1}{\partial x_1^2} - 3(1+Bi)T_1 = \\ = \frac{h^2(1+\varepsilon_0)}{a_T} \cdot \frac{\partial T_1}{\partial t_1} - \frac{3Bi}{2}(T_c^+ - T_c^-) - \frac{\varepsilon_0 h^3}{\alpha_T a_T} \cdot \frac{1-2\nu}{1+\nu} \cdot \frac{\partial^3 W_1}{\partial t_1 \partial x_1^2}. \tag{4}$$

The last term in Eq. (4) characterizes thermoelastic dissipation of mechanical energy, and thus, Eq. (4) together with Eq. (2) forms a coupled system representing the thermoelasticity problem for plates. Here $Bi = \dfrac{h\lambda_c}{\lambda_q}$ is a Biot coefficient of heat exchange with environment, $a_T = \dfrac{\lambda_q}{c_\varepsilon}$ is thermal diffusivity coefficient, $\varepsilon_0 = \dfrac{c_\sigma - c_\varepsilon}{3c_\varepsilon} \cdot \dfrac{1+\nu}{1-\nu}$ characterizes the thermoelastic dissipation of the material (from thermodynamic considerations $\varepsilon_0 > 0$), c_ε is the heat capacity at constant deformation, c_σ is the heat capacity at constant stress. Note that by assuming $\varepsilon_0 = 0$ we obtain the well-known heat conduction equation for a thin plate with heat exchange with the external environment. The heat conduction Eq. (4) corresponds to linear temperature distribution in a thin plate [2,3]

$$T_* = T_0(x_1,t_1) + \frac{z_1}{h} T_1(x_1,t_1), \tag{5}$$

where the integral characteristic $T_0 = \dfrac{1}{2h}\displaystyle\int_{-h}^{h} T_* dz_1$ is the temperature of the midplane of the plate $z_1 = 0$. Let a constant temperature be maintained on the boundary surfaces $x_1 = 0, x_1 = a$,

$$T_1 = 0 \quad \text{when} \quad x_1 = 0,\ x_1 = a. \tag{6}$$

Taking the initial conditions for the functions W_1 and T_1 in the form

$$W_1 = \varphi_1(x_1), \quad \frac{\partial W_1}{\partial t_1} = \psi_1(x_1), \quad T_1 = 0 \quad \text{when} \quad t_1 = 0. \tag{7}$$

From the perspective of mathematical modeling, a controlled system is defined as an entity, whose temporal behavior can be manipulated by choosing external thermal inputs. The peculiarity of controlling thermoelastic vibrations in this problem is taken into account when heat conduction, thermal deformations, and stresses in the elastic plate are induced by heat exchange with the surrounding environment and are also influenced by the deformation process itself.

Let's assume that the function characterizing the temperature difference of the external environment on the planes $z = \pm h$ is represented as

$$\frac{\alpha_T}{2}(T_c^+ - T_c^-) = v(x_1)u(t_1), \tag{8}$$

$u(t_1)$ represents the control, the function characterizing the change in temperatures over time. There is the possibility to control the shape of the external temperature field source; $v(x_1)$ is the distribution function on the surfaces of the plate. The control problem is to establish, at a certain moment in time τ, the position of the thermoelastic system that is closest to the temperature quasi-static state, taking into account thermoelastic dissipation of mechanical energy.

The task is set to translate the thermoelastic process of plate vibrations described by Eqs. (2), (4), and conditions (3), (6), (7), (8) into a quasi-static mode.

$$\frac{\partial^2 M_x}{\partial x_1^2} = 0,$$
$$h^2 \frac{\partial^2 T_1}{\partial x_1^2} - 3(1+Bi)T_1 = \frac{h^2(1+2\varepsilon_0(1-\nu))}{a_T} \cdot \frac{\partial T_1}{\partial t_1} - 3Biv(x_1)u(t_1), \tag{9}$$
when $t_1 = \tau$.

In this context, the functional characterizing the energy of the external temperature influence attains its minimum value

$$\Phi = \int_0^\tau u^2(t_1)dt + 2\int_0^a v^2(x_1)dx \tag{10}$$

in the space $L_2(0,\tau)xL_2(0,a)$.

Mathematical Methods and Solution. Adopting a set of new dimensionless variables $x = \frac{x_1}{a}, t = \frac{t_1}{\tau}, W = \frac{W_1}{h}, T = \alpha_T T_1$, where $x \in [0;1], t \in [0;1]$. In this case, the vibration Eq. (2) under the influence of the temperature field takes the form

$$\frac{\partial^4 W}{\partial x^4} + \frac{a^2}{h^2}(1+\nu)\frac{\partial^2 T}{\partial x^2} + \frac{1}{\omega^2} \cdot \frac{\partial^2 W}{\partial t^2} = 0, \tag{11}$$

where $\omega = \tau\omega_0$, $\omega_0^2 = \frac{Eh^2}{3\rho a^4(1-\nu^2)}$. The heat conduction Eq. (4) taking into account condition (8), is represented in the form

$$\frac{h^2}{a^2} \cdot \frac{\partial^2 T}{\partial x^2} - 3(1+Bi)T = \tau_0 \frac{\partial T}{\partial t} - 3Biu(t)v(x) - \frac{\varepsilon h^2 \tau_0}{a^2(1+\nu)} \cdot \frac{\partial^3 W}{\partial t \partial x^2}, \tag{12}$$

where $\tau_0 = \frac{h^2(1+\varepsilon_0)}{\tau a_T}$, $\varepsilon = \frac{\varepsilon_0(1-2\nu)}{(1+\varepsilon_0)}$.

The boundary and initial conditions (3), (6), (7) will be

$$W = 0, \quad \frac{\partial^2 W}{\partial x^2} = 0, \quad T = 0 \quad \text{when} \quad x = 0,\ x = 1, \tag{13}$$

$$W = \varphi(x), \quad \frac{\partial W}{\partial t} = \psi(x), \quad T = 0 \quad \text{when} \quad t = 0. \tag{14}$$

For functions $\varphi(x), \psi(x)$ there are continuity requirements along with their derivatives of the appropriate order, as well as conditions for matching the initial and boundary conditions.

The control problem of the thermoelastic process consists of transforming the system (11), (12) with conditions (13), (14) into equations of the quasi-static thermoelastic problem.

$$\frac{\partial^4 W}{\partial x^4} + \frac{a^2}{h^2}(1+\nu)\frac{\partial^2 T}{\partial x^2} = 0,$$
$$\frac{h^2}{a^2}\cdot\frac{\partial^2 T}{\partial x^2} - 3(1+Bi)T = \tau_1\frac{\partial T}{\partial t} - 3Biu_1 v(x), \quad \text{when} \quad t \geq 1, \tag{15}$$

here $u_1 = u(1), \tau_1 = \tau_0(1+\varepsilon), T(t=\infty) = Const$. During the time τ, i.e. at $t=1$, the functional attains its minimum value in $L_2(0,1)xL_2(0,1)$

$$\Phi = \int_0^1 u^2 dt + 2\int_0^1 v^2 dx. \tag{16}$$

It should be noted that $\tau_0 \cdot \tau$ (or $\tau_1 \cdot \tau$) is the characteristic time for heating the plate through its thickness in the dynamic thermoelastic process (or in the quasi-static process). Thermal diffusivity a_T characterizes the rate, at which the temperature of the material equalizes in non-equilibrium heat processe [2,6]. By taking $\varepsilon_0 = 0$, we obtain the equations for elastic transverse vibrations of the plate-strip under a temperature field arising from heat exchange with the external environment. For solid, isotropic bodies $0 < \varepsilon \ll 1$.

We represent the solutions of Eqs. (11), (12) in the form

$$W(x,t) = \sum_{m=1}^{\infty} \eta_m(t)\sin\pi mx,$$
$$T(x,t) = \textstyle\sum_{m=1}^{\infty} \vartheta_m(t)\sin\pi mx, \tag{17}$$

that satisfies conditions (13). We also consider expansions of the functions

$$v(x) = \sum_{m=1}^{\infty} v_m \sin\pi mx, \tag{18}$$

$$\varphi(x) = \sum_{m=1}^{\infty} \varphi_m \sin\pi mx, \quad \psi(x) = \sum_{m=1}^{\infty} \psi_m \sin\pi mx.$$

To determine the function $\eta_m(t)$, $\vartheta_m(t), m = 1,2,\ldots,$ we obtain the following coupled system of equations:

$$\frac{d^2\eta_m(t)}{dt^2} + \omega_m^2\eta_m(t) - \frac{a^2}{h^2}\cdot\frac{1+\nu}{\pi^2 m^2}\omega_m^2\vartheta_m(t) = 0,$$
$$\tau_0\frac{d\vartheta_m(t)}{dt} + A_m\tau_0\vartheta_m(t) + \varepsilon\frac{h^2}{a^2}\cdot\frac{\tau_0\pi^2 m^2}{1+\nu}\cdot\frac{d\eta_m(t)}{dt} = 3v_m Biu(t)$$

or

$$\left(\frac{1}{\omega_m^2}\cdot\frac{d^2}{dt^2}+1\right)\left(\frac{d\eta_m(t)}{dt} + A_m\eta_m(t)\right) + \varepsilon\frac{d\eta_m(t)}{dt} = B_m v_m u(t), \tag{19}$$

$$\vartheta_m(t) = \frac{h^2\pi^2 m^2}{a^2(1+\nu)}\left(\eta_m(t) + \frac{1}{\omega_m^2}\cdot\frac{d^2\eta_m(t)}{dt^2}\right), \tag{20}$$

$$\omega_m = \omega\pi^2 m^2, \quad A_m = \frac{3(1+Bi)}{\tau_0} + \frac{h^2\pi^2 m^2}{\tau_0 a^2}, \quad B_m = \frac{3Bi(1+\nu)a^2}{\tau_0 h^2\pi^2 m^2}.$$

And for v_m we will have

$$v_m = 2\int_0^1 v(x)\sin\pi mx dx. \tag{21}$$

To satisfy the required conditions when $t \geq 1$, we obtain

$$\pi^2 m^2 \eta_m(t) = \frac{a^2}{h^2}(1+\nu)\vartheta_m(t),$$

$$(1+\varepsilon)\frac{d\vartheta_m(t)}{dt} + A_m\vartheta_m(t) - 3\frac{Bi}{\tau_0}u_1 v_m = 0,$$

what can also be represented in the form of

$$\frac{d^2\eta_m(t)}{dt^2} = 0, \quad (1+\varepsilon)\frac{d\eta_m(t)}{dt} + A_m\eta_m(t) = B_m v_m u_1 \quad \text{when} \quad t = 1. \tag{22}$$

The unique solution to the problem (19), (20), (14) is presented in the form

$$\eta_m(t) = e^{-h_m t}(c_m\cos\Omega_m t + d_m\sin\Omega_m t) + f_m e^{-\lambda_m t} + B_m\omega_m^2 v_m G_m(t), \tag{23}$$

where

$$c_m = \frac{(\omega_m^2 + \lambda_m^2 - 2h_m\lambda_m)\varphi_m - 2h_m\psi_m}{(h_m-\lambda_m)^2 + \Omega_m^2},$$

$$d_m = \frac{\varphi_m((\lambda_m - h_m)(\lambda_m h_m - \omega_m^2) + \lambda_m\Omega_m^2) + \psi_m(\lambda_m^2 - h_m^2 + \Omega_m^2)}{\Omega_m((h_m-\lambda_m)^2 + \Omega_m^2)}, \tag{24}$$

$$f_m = \frac{\varphi_m(h_m^2 - \omega_m^2 + \Omega_m^2) + 2h_m\psi_m}{(h_m-\lambda_m)^2 + \Omega_m^2},$$

$$G_m(t) = \int_0^t F_m(t-s)u(s)ds, \tag{25}$$

and

$$F_m(t) = \frac{1}{(h_m-\lambda_m)^2 + \Omega_m^2}\left[e^{-\lambda_m t} - e^{-h_m t}\left(\cos\Omega_m t + \frac{h_m-\lambda_m}{\Omega_m}\sin\Omega_m t\right)\right], \tag{26}$$

$$v_m = 2\int_0^1 v(x)\sin\pi mx dx, \quad \varphi_m = 2\int_0^1 \varphi(x)\sin\pi mx dx, \quad \psi_m = 2\int_0^1 \psi(x)\sin\pi mx dx. \tag{27}$$

The characteristic equation

$$\left(\frac{s^2}{\omega_m^2} + 1\right)(s + A_m) + \varepsilon s = 0 \tag{28}$$

for transverse thermoelastic vibrations of a thin, isotropic plate, from thermodynamic considerations (as $\varepsilon_0 \ll 1$), has one negative solution and complex solutions $-\lambda_m,\ -h_m \pm i\Omega_m,\ \lambda_m > 0,\ \Omega_m > 0,\ h_m \geq 0$ for all $m = 1,2,\ldots$ When $\varepsilon_0 = 0$ and thermoelastic dissipation of mechanical energy is not considered $\lambda_m = A_m$, $h_m = 0,\ \Omega_m = \omega_m$. For the functional (16) we will have

$$\Phi = \sum_{m=1}^{\infty} v_m^2 + \int_0^1 u^2(t)dt, \tag{29}$$

and represent function $u(t)$ in the form

$$u(t) = \sum_{k=1}^{\infty} (a_k \cos \omega_k t + b_k \sin \omega_k t) + a_0.$$

The control function $v(x)u(t)$ is divided into two components, and to find the optimal control, momentary relationships and nonlinear equations are derived [1].

Note that for the solutions of the characteristic Eq. (28), we have

$$\lambda_m + 2h_m = A_m,$$
$$\Omega_m^2 + 2h_m\lambda_m + h_m^2 = \omega_m^2(1+\varepsilon),$$
$$\lambda_m(h_m^2 + \Omega_m^2) = A_m\omega_m^2,$$

when $t > 1$

$$\vartheta_m(t) = e^{-\frac{A}{1+\varepsilon}(t-1)}\vartheta_m(1) - 3\frac{Bi}{\tau_0 A_m}u_1 v_m(e^{-\frac{A}{1+\varepsilon}(t-1)} - 1),$$
$$\vartheta_m(1) = \frac{\pi^2 m^2 h^2}{a^2(1+\nu)}\eta_m(1).$$

Let's assume that $\tau = \dfrac{2\alpha}{\omega_0\pi}$, $\alpha = 1,2,\ldots$, is a multiple of the vibration period [1]. The system is controllable for $\tau \geq \dfrac{2}{\omega_0\pi}$, if the representation coefficients (23) c_m, d_m, f_m rapidly decrease with respect to $m \to \infty$, besides

$$c_m + f_m = \varphi_m,$$

$$\Omega_m d_m - h_m c_m - \lambda_m f_m = \psi_m.$$

In this case

$$\Phi_{1m}(v_m, a_m, b_m) = \left.\frac{d^2\eta_m}{dt^2}\right|_{t=1} = 0,$$
$$\Phi_{2m}(v_m, a_m, b_m) = (1+\varepsilon)\frac{d\eta_m(t)}{dt} + A_m\eta_m(t) - B_m v_m u(1) = 0. \tag{30}$$

The minimizing functional (29) takes the form

$$a_0 = 0, \quad \Phi = \sum_{m=1}^{\infty} v_m^2 + \frac{1}{2}\sum_{m=1}^{\infty}(a_m^2 + b_m^2). \tag{31}$$

A known method of minimizing the functional is used, resulting in a system of algebraic equations to determine a_m, b_m, v_m and the multipliers $\delta_m, \mu_m,\ m = 1, 2, \cdots$:

$$\begin{aligned} &\Phi_{1m}(v_m, a_m, b_m) = 0, \\ &\Phi_{2m}(v_m, a_m, b_m) = 0, \\ &2v_m + \delta_m \frac{\partial \Phi_{1m}}{\partial v_m} + \mu_m \frac{\partial \Phi_{2m}}{\partial v_m} = 0, \\ &a_m + \delta_m \frac{\partial \Phi_{1m}}{\partial a_m} + \mu_m \frac{\partial \Phi_{2m}}{\partial a_m} = 0, \\ &b_m + \delta_m \frac{\partial \Phi_{1m}}{\partial b_m} + \mu_m \frac{\partial \Phi_{2m}}{\partial b_m} = 0. \end{aligned} \tag{32}$$

Conclusion. The second variation of the functional

$$\Phi + \sum_{m=1}^{\infty} (\delta_m \Phi_{1m} + \mu_m \Phi_{2m})$$

is positive, therefore, extreme solutions lead to a minimum of the functional.

The representation of the control function, i.e., the function of the external environment temperature, in the form of Eq. (8), leads to a system of nonlinear algebraic equations for finding the optimal control function, as well as the vibration and temperature functions.

The solution of the system of algebraic equations is determined ambiguously. Having one of the solutions, the functional takes the same values on other solutions. Freedom in choosing controls is used to highlight the solution that is the best from the point of view of practice.

Received 25.01.2024
Reviewed 20.02.2024
Accepted 29.02.2024

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ՍԱԼ-ՇԵՐՏԻ ՋԵՐՄԱԱՌԱՁԳԱԿԱՆ ՏԱՏԱՆՈՒՄՆԵՐԻ ՕՊՏԻՄԱԼ ՂԵԿԱՎԱՐՄԱՆ ՄԱՍԻՆ

Դիտարկվում է ղեկավարվող, բաշխված պարամետրով համակարգի համար օպտիմալ ղեկավարման խնդիր: Բարակ, իզոտրոպ սալերում ոչ ստացիոնար, դինամիկական ջերմաառաձգականության պրոցեսը նկարագրվում է ջերմաստիճանային դաշտում լայնական փտտանումների և սալի համար ջերմահաղորդականության դիֆերենցիալ հավասարումների համակարգով: հաշվի է առնվում մեխանիկական էներգիայի ջերմաառաձգական ցրումը, որի պատճառով դիտարկվում է ջերմաառաձգականության կապակցված խնդիր: Սալի դիմային հարթությունների և արտաքին միջավայրի միջև տեղի ունի ջերմափոխանակություն: Խնդիր է դրված ջերմաառաձգականության դիտարկվող պրոցեսը որոշակի ժամանակի ընթացքում բերել քվազիստատիկ վիճակի: Ընդ որում արտաքին ջերմային ազդեցությունը բնութագրող ֆունկցիոնալը հասնում է փոքրագույն արժեքի, իսկ արտաքին միջավայրի ջերմաստիճանը ղեկավարող ֆունկցիան է:

С. А. ДЖИЛАВЯН, Э. Р. ГРИГОРЯН

ОБ ОПТИМАЛЬНОМ УПРАВЛЕНИИ ТЕРМОУПРУГИМИ КОЛЕБАНИЯМИ ПЛАСТИНКИ-ПОЛОСЫ

Рассматривается задача оптимального управления для управляемой системы с распределенными параметрами. Процесс нестационарной, динамической термоупругости в тонких, изотропных пластинках описывается системой дифференциальных уравнений поперечных колебаний в температурном поле и теплопроводности для пластинки. Учитывается термоупругое рассеяние механической энергии, что приводит к связанной задаче термоупругости между плоскостями пластинки и окружающей средой, где осуществляется теплообмен. Ставится задача перевода рассматриваемого процесса термоупругости за некоторое время в квазистатический режим. При этом функционал, характеризующий энергию внешнего теплового воздействия, достигает наименьшего значения, а температура окружающей среды является управляемой функцей.