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Mathematics

"BONUS HUNGER" BEHAVIOR IN THE ALTERNATIVE BONUS–MALUS SYSTEM

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In this article the "bonus hunger" behavior for the Alternative bonus–malus system (BMS) is discussed. The Alternative BMS is a model, where the next premium is the combination of the previous premium and the aggregate claim amount. The key characteristics for the comparison are the discounted premium reduction for some time horizon and the entire claim amount. Existence of the steady state for the BMS discussed in this paper was proved and the probability of claiming for the general model and for its steady state was found out.

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Introduction. One of the consequences of the BMS is the willingness of the insured to overtake the small claims on its own debit and not claim them, to keep the reduced premium payment. This problem in 1960 C. Philipson called "hunger for bonus". Alting von Geusau investigates "to what extent it is possible to develop a theoretical framework to test that a no-claim-discount-system will prevent the insured from submitting small claims to the insurance company", and "that the insured who has just lost his no-claim discount will use every possibility for submitting claims with in his mind the idea that in this way he will earn back his higher non-reduced premium" [1]. U. Grenander derives equations to determine a rule of the form "pay the damage, if its amount is smaller than a critical value and claim it otherwise". However, the equations are generally difficult to solve, and it is not proved that they really determine an optimal policy in the sense that the total expected discounted cost of premiums and payments during a long future planning period is minimized. A. Martin-Lof shows that a decision rule of the form formulated by Grenander is optimal in the sense that it minimizes the total expected costs. The decision rule is derived by applying the general theory of Markov decision processes, which find an optimal control iteratively by using dynamic programming. In that work, however,

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the analysis was restricted to the case, where the policyholder takes a decision only at the end of an insurance period for the total amount of damage sustained during that insurance period.

Haehling von Lanzenauer and Lundberg develop a model, which can be used in deriving the distribution of the number of claims for insurances with merit-rating structures. The problem is formulated and solved as a regular Markov process with the claim behavior integrated in the analysis. Haehling von Lanzenauer develops an optimal decision rule for situations, where the policyholder takes a decision more than once a year, which is valid for any merit-rating system. He splits up a year into a number of periods, which results in a discrete model in which the optimal critical claim size can be determined by dynamic programming. However, this derivation of an optimal critical claim size is incomprehensible. Lemaire derives an algorithm for obtaining the optimal strategy for a policyholder. In his model the policyholder remains always insured (the so-called infinite horizon model), which leads to a critical claim size which is independent of the year in which the accident takes place. Also, in order to compute the optimal policy, he uses policy iteration, which is very time-consuming, whenever the state space is large. He applies this algorithm in [2] to compare BMS used in Norway, Denmark, Finland, Sweden, Switzerland and West Germany.

Some insurance companies actually enable policyholders to buy the bonus by paying extra premium. Thus the bonus–malus system becomes also a strong marketing instrument.

While analyzing the effect of “ bonus hunger” it is necessary to answer the following question: “When the policyholder will not report the claim?” The answer is: “He will not report the claim, if its amount is less than discounted value of all future premium reductions” [3]. The premium reductions are clearly given by difference between premiums.

Let us consider a BMS introduced in [4]. Suppose that a series of independent and identically distributed random variables $Y_1, Y_2, \dots$ are yearly aggregate claims of a policyholder, given on a $(\Omega, \mathcal{F}, (\mathcal{F}_n)_{n\geq 0}, \mathrm{P})$ filtered probability space, where $\mathcal{F}_0 = \{\varnothing, \Omega\}$ and $\mathcal{F}_n = \sigma\{Y_1, Y_2, \dots Y_n\}$. And suppose that $Y_1, Y_2, \dots$ random variables are so that $EY < \infty$ condition is satisfied. The next premium is defined by

$$P_n = (1-\alpha_n)P_{n-1} + \beta_n Y_n, \qquad n \geq 1, \tag{1}$$

where P_n is the premium of a policyholder for the n-th year in the BMS; Y_n is an aggregate claim loss for the given policyholder for $(n-1; n)$ time interval (it is necessary to note that Y_n is independent of P_{n-1} for all n, $n \geq 1$). $\alpha_n \in (0,1)$ called a series of bonus factors and $\beta_n \in (0,1)$ called a series of malus factors.

Probability of Claiming. Consider a policyholder, who is in the portfolio for $n-1$ years and has just had an accident. We denote P'_{n+k} for a premium after k years in the case that the policyholder will not claim this accident and will not have any accident during next k years. Using formula (1) for $Y_j = 0,\ j = n,\ n+1, \dots,\ n+k$, we get:

$$P_n^{'} = (1-\alpha_n)P_{n-1},$$
$$P_{n+1}^{'} = (1-\alpha_{n+1})P_n^{'} = (1-\alpha_{n+1})(1-\alpha_n)P_{n-1},$$
$$\dots$$
$$[P_{n+k}^{'} = (1-\alpha_{n+k})P_{n+k-1}^{'} = (1-\alpha_{n+k})\dots(1-\alpha_n)P_{n-1} = P_{n-1}\prod_{j=0}^{k}(1-\alpha_{n+j}).$$

Let $P_{n+k}^{''}$ be a premium after k years in the case that the policyholder claim this accident and will not have any accident during next k years. Applying (1) for this premium, where $Y_n > 0$ and $Y_j = 0,\ j = n+1,\dots,n+k$, we have:

$$P_n^{''} = (1-\alpha_n)P_{n-1} + \beta_n Y_n,$$
$$P_{n+1}^{''} = (1-\alpha_{n+1})P_n^{''} = (1-\alpha_{n+1})(1-\alpha_n)P_{n-1} + (1-\alpha_{n+1})\beta_n Y_n,$$
$$\dots$$
$$P_{n+k}^{''} = (1-\alpha_{n+k})P_{n+k-1}^{''} = (1-\alpha_{n+k})\dots(1-\alpha_n)P_{n-1} +$$
$$+(1-\alpha_{n+k})\dots(1-\alpha_{n+1})\beta_n Y_n =$$
$$= P_{n-1}\prod_{j=0}^{k}(1-\alpha_{n+j}) + \beta_n Y_n \prod_{j=1}^{k}(1-\alpha_{n+j}).$$

The discounted premium reduction for k years is

$$u_n = \sum_{m=0}^{k}\left(P_{n+m}^{''} - P_{n+m}^{'}\right)v^m = \beta_n Y_n + \sum_{m=1}^{k}\left(P_{n+m}^{''} - P_{n+m}^{'}\right)v^m =$$
$$= \beta_n Y_n\left(1 + \sum_{m=1}^{k} v^m \prod_{j=1}^{m}(1-\alpha_{n+j})\right),$$

where v is the discount factor.

The probability of claiming is

$$p_n(u_n) = P(Y_n > u_n) = P\left(1 > \beta_n\left(1 + \sum_{m=1}^{k} v^m \prod_{j=1}^{m}(1-\alpha_{n+j})\right)\right).$$

It can be concluded that for the discussed model the probability of claiming does not depend on claim amount. It depends on the relationship between bonus and malus coefficients, discount factor, as well as the future time horizon.

Propensity to Claim for the Steady State. When the system is stabilized we say that the BMS is in the steady state. The following Lemma shows that for the BMS introduced in [4] there exists a steady state.

L e m m a 1. The coefficients α_n and β_n, given by the formulas

$$\alpha_n = \left|\frac{Y_c - P_{n-1}}{F_Y^{-1}(\varepsilon) - EY}\right| \cdot \frac{EY}{P_{n-1}},$$
$$\beta_n = \left|\frac{Y_c - P_{n-1}}{F_Y^{-1}(\varepsilon) - EY}\right|,$$

have finite limits as $n \to \infty$.

Here Y_c is the critical value of aggregate claim for a given ε satisfying to the condition $P(P_n > Y_c) = 1 - \varepsilon$ (see [4]).

P r o o f . The nonnegative martingale $(P_n, \mathfrak{F}_n)$ given with (1) satisfies to the conditions of Doob's theorem on submartingales and its Corollary (see [5], p. 688), so there exists finite $\lim_{n\to\infty} P_n = P_\infty$ a.s. It is not difficult to see that $|Y_c - P_\infty| < \infty$, and for sufficient small ε the condition $F_Y^{-1}(\varepsilon) - EY \neq 0$ is satisfied. So, we have

$$\lim_{n\to\infty} \beta_n = \lim_{n\to\infty} \left| \frac{Y_c - P_{n-1}}{F_Y^{-1}(\varepsilon) - EY} \right| = \left| \frac{Y_c - P_\infty}{F_Y^{-1}(\varepsilon) - EY} \right| \triangleq \beta < \infty$$

and

$$\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \left| \frac{Y_c - P_{n-1}}{F_Y^{-1}(\varepsilon) - EY} \right| \cdot \frac{EY_n}{P_{n-1}} = \left| \frac{Y_c - P_\infty}{F_Y^{-1}(\varepsilon) - EY} \right| \cdot \frac{EY}{P_\infty} \triangleq \alpha < \infty. \quad \square$$

Now consider the BMS model (1) at the steady state. This means that starting from some time t the bonus and malus coefficients will not depend on time and we can consider the following premium model

$$P_k = (1-\alpha) P_{k-1} + \beta Y_k.$$

In this case we have:

$$u_n = \beta Y_n \left(1 + \sum_{m=1}^{k} v^m \prod_{j=1}^{m} (1-\alpha) \right) = \beta Y_n \frac{1 - v^{k+1}(1-\alpha)^{k+1}}{1 - v(1-\alpha)}.$$

Now we state the problem as follows: how many years at least it needs to not have an accident for getting as much discounted premium reduction as the current cost of the accident. We need to solve the following equation with respect to k:

$$Y_n = u_n.$$

The result is

$$k^* = \log_{v(1-\alpha)} \frac{\beta - 1 + v(1-\alpha)}{\beta} - 1.$$

Then the probability of claiming is $P(k > k^*)$.

Conclusion. In the paper it is discussed the "bonus hunger" phenomenon for an insurance policy, where the next premium is defined by the Alternative BMS. The probability of claiming was found out for the general model and for its steady state. For both cases it can be concluded that the probability of claiming does not depend on claim amount. For the steady state the time of return of the cost of claim was found out for a policyholder, who has just had an accident.

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